Group Theory Quiz I'm flipping out over this quiz!!! Jason Hagen
Analysis 2013-14 Deggeller/Hahn 29/34

1. Indicate whether each of the following sets of numbers constitutes a mathematical group. If your answer is no, explain why it is not a group. [2 each]

a) Rational numbers under addition yes

b) Nonzero real numbers under multiplication yes

c) Rational powers of 10 under multiplication $10^2, 10^{1/2}, 10^{3/2}$ etc.... yes

d) Rational powers of 10 under addition no
$10^0 + 10^1$ would equal 11, which is not part of the group.

2. Explain why the trivial set of numbers: { 1 , -1} is a group under multiplication but not addition. [4]
It is not a group under addition because $1+(-1)$ is 0, which is not included in the group. It is a group under multiplication because $1 \cdot 1$ is 1, $1 \cdot (-1)$ is -1, and $(-1) \cdot (-1)$ is 1, which are all contained in the group.

3. a) How many elements are there in a 4 post snap group? 24 [2]

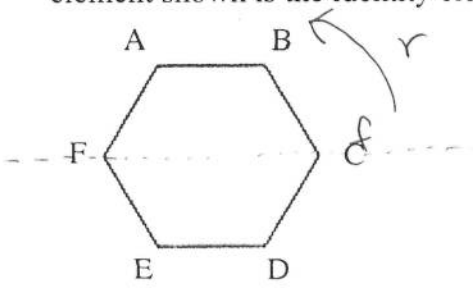
$4! = 24$

b) which geometric ~~rotation~~ reflection group was isomorphic to the 4 post snap group?

tetrahedral [2]

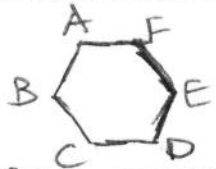
4. Prove that the size of the set of natural numbers is the same as the size of the set of positive odd numbers. Be very clear. [3]
Natural numbers: $1, 2, 3, 4, 5, 6, \ldots n_k, n_{k+1}$
Positive odd numbers: $1, 3, 5, 7, 9, 11, \ldots O_k, O_{k+1}$ $\rightarrow O_k = 2n_k - 1$
They are the same size because with every natural number, there will be an odd number that is one less than twice the natural number, showing a one-to-one correlation.

5. Consider the dihedral (reflection/rotation) group for the regular hexagon where the element shown is the identity element.



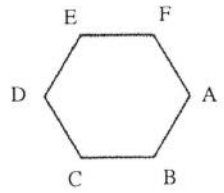
Let "f" be defined as the operation "reflect over the horizontal symmetry line (for the identity this would be line FC). Let "r" be defined as the operation rotate counterclockwise by 60 degrees.

a) Draw the element represented by $f \bullet r^2$ [2]



b) ~~Using no more than three operations,~~ Name the element below (use r's and f's). [2]

two diff. ways



r^4, fr^2f

6. Consider the rotation group of a regular nonagon (9 sided figure).

a. Explain why R_{40}, and R_{80} could be generators of this group but R_{120} would not be. [3]

R_{120} is the same as R_3, and if R_3 is used as a generator, only R_3 and R_9 can be reached, and the period is 3. R_{40} and R_{80} are the same as R_4 and R_2 respectively, and are both relatively prime with 9, so their period is 9 and it can serve as a generator. (-1)

b. What would the period of R_{80} be? 9 [2]

c. How many elements are there in the described group? 9 [2]

d. How many elements would there be in the **reflection** group for the regular nonagon?

18 [2]

e. How many elements would there be in the reflection group of a regular nonagonal prism? ~~10~~ [2] 36

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