[illegible] and math make an infinite group of awesomeness

Per: #3 Date: 2/12/14

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Analysis – Deggeller/Hahn
Unit 7 Quiz 2

1. Write out the first 5 rows of Pascal's Triangle, and then use it to expand $(x+y)^5$

$$\begin{matrix} & & & & & 1 & & & & & \\ & & & & 1 & & 1 & & & & \\ & & & 1 & & 2 & & 1 & & & \\ & & 1 & & 3 & & 3 & & 1 & & \\ & 1 & & 4 & & 6 & & 4 & & 1 & \\ 1 & & 5 & & 10 & & 10 & & 5 & & 1 \end{matrix}$$

$$\boxed{x^5 + 5x^4y + 10x^3y^2 + 10x^2y^3 + 5xy^4 + 1}$$

2. Use your result from #1 to find a trig identity for $\sin(5\theta)$ in terms of $\cos\theta$ and $\sin\theta$.

$$\text{cis}\,5\theta = (\text{cis}\,\theta)^5 = (\cos\theta + i\sin\theta)^5 = \cos^5\theta + 5i\cos^4\theta\sin\theta + 10\overset{-1}{\cancel{i^2}}\cos^3\theta\sin^2\theta + 10\overset{-i}{\cancel{i^3}}\cos^2\theta\sin^3\theta$$

$$+ 5\cancel{i^4}\cos\theta\sin^4\theta + \cancel{i^5}\sin^5\theta$$

$$= \cos^5\theta - 10\cos^3\theta\sin^2\theta + 5\cos\theta\sin^4\theta + i\underline{(5\cos^4\theta\sin\theta - 10\cos^2\theta\sin^3\theta + \sin^5\theta)}$$

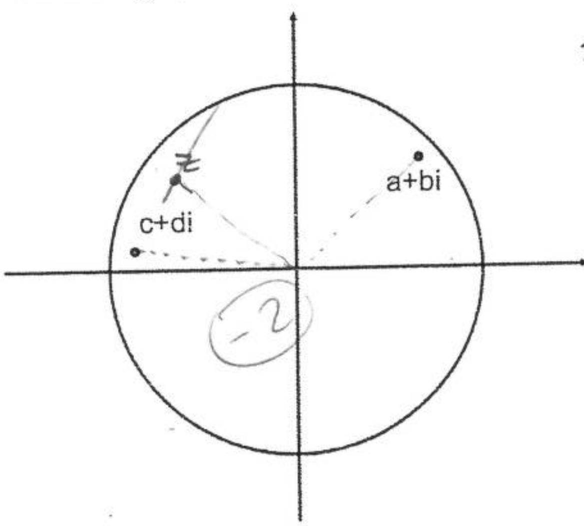
$$\boxed{\sin 5\theta = \sin^5\theta - 10\cos^2\theta\sin^3\theta + 5\cos^4\theta\sin\theta}$$

3. Simplify: $(-1+i)^{30}$. Give your answer in $a+bi$ form.

$$\left(\sqrt{2}\,\text{cis}\,\tfrac{3\pi}{4}\right)^{30} = 2^{15}\,\text{cis}\,\tfrac{45}{2}\pi = 2^{15}\,\text{cis}\,\tfrac{\pi}{2} = 2^{15}(0+i)$$

$$= \boxed{32768i}$$

4. The circle below is a unit circle, and $z = (a+bi)(c+di)$. Plot z on the graph as accurately as possible. On the right, include a written explanation of why you placed z where you did.



$$z = \underbrace{(ac - bd)}_{x\text{ coor}} + i\underbrace{(ad+bc)}_{y\text{ coor}}$$

ac-bd is negative because ac is negative and bd is positive. ad+cd is also negative because even though ad is positive, it is relatively small and bc is negative.

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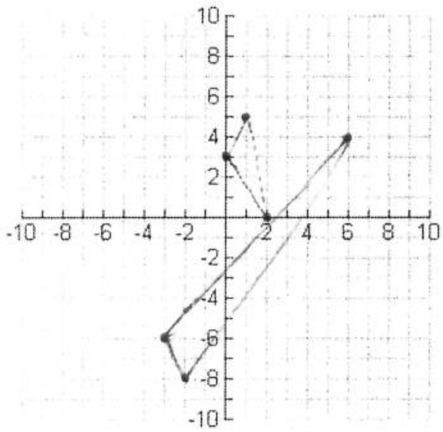
5. Given the matrix transformation $\begin{bmatrix} 3 & -1 \\ 2 & -2 \end{bmatrix}\begin{bmatrix} 2 & 1 & 0 \\ 0 & 5 & 3 \end{bmatrix} = A\ldots$

a) Solve for matrix A

$$\boxed{A = \begin{bmatrix} 6 & -2 & -3 \\ 4 & -8 & -6 \end{bmatrix}}$$

b) Graph the pre-image (connected with dotted lines) and the image of the transformation (connected with solid lines) on the graph below.



6. Write a single matrix that would transform a pre-image according to the following (in order):
   i. flip over the y-axis $\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$
   ii. rotate counterclockwise by 30 degrees $\begin{bmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$
   iii. stretch in the x-direction by a factor of 2. $\begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}$

$$\begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} -\frac{\sqrt{3}}{2} & -\frac{1}{2} \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix} = \boxed{\begin{bmatrix} -\sqrt{3} & -1 \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}}$$

7. a) Write a single matrix that would transform a pre-image according to the following (in order):
   i. shear in the y-direction by a factor of 3. $\begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$
   ii. Translate 5 units in the x-direction and 2 units in the y-direction. $\begin{bmatrix} 1 & 0 & 5 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$

$$\begin{bmatrix} 1 & 0 & 5 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \boxed{\begin{bmatrix} 1 & 0 & 5 \\ 3 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}}$$

c) Show how you would use your matrix from part (a) to find the image of the point (j,k) by matrix multiplication.

$$\begin{bmatrix} 1 & 0 & 5 \\ 3 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} j \\ k \\ 1 \end{bmatrix} = \begin{bmatrix} j+5 \\ 3j+k+2 \\ 1 \end{bmatrix} \leftarrow \text{so the point is now } (j+5,\ 3j+k+2)$$

There is an additional 1 in the point matrix so the point can translate. When finished multiplying the matrices, you disregard the

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