Vector Quiz 2 Analysis 2013-14 30/30 Vector Inspector [illegible]

No Calculators

1. Given 3-d points P = (-2, 3, 5) and Q = (2, 6, 5).

a) Find vector PQ = < 4 , 3 , 0 > [2]

b) Find the parametric equation of line PQ. [3]

$x = -2 + \frac{4}{5}t$

$y = 3 + \frac{3}{5}t$

$z = 5$

c) Find the midpoint of segment PQ. [2]

$\frac{-2+2}{2} = 0 \quad \frac{6+3}{2} = \frac{9}{2} \quad \frac{5+5}{2} = 5$ $\boxed{(0, \frac{9}{2}, 5)}$

d) Find point R that is on line PQ, but 7 units away from point P (in the opposite direction of Q). [3]

$x = -2 + \frac{4}{5}(-7) = -\frac{38}{5}$

$y = 3 + \frac{3}{5}(-7) = -\frac{6}{5}$

$z = 5$

$\boxed{(-\frac{38}{5}, -\frac{6}{5}, 5)}$

e) Find the equation of the sphere with center P where point Q is on the sphere. [2]

$\boxed{(x+2)^2 + (y-3)^2 + (z-5)^2 = 25}$

f) Find the scalar projection of vector PQ on the y axis. [3]

y axis: $\langle 0, 1, 0 \rangle$

$\frac{3}{1}\langle 0, 1, 0 \rangle = \langle 0, 3, 0 \rangle$

$\boxed{3}$

g) Consider a third point T= (-3, k, 4). Find k such that vectors PQ and PT are orthogonal. [3]

$PQ = \langle 4, 3, 0 \rangle$

$PT = \langle -1, k-3, -1 \rangle$

$\cos 90 = \frac{-4 + 3k - 9 + 0}{\sqrt{25}\sqrt{2 + (k-3)^2}}$

$0 = \frac{-4 + 3k - 9}{\sqrt{25}\sqrt{2 + (k-3)^2}}$

$3k = 13$

$\boxed{k = \frac{13}{3}}$

$PT = \langle -1, \frac{4}{3}, -1 \rangle$

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2. Find the equation of the plane passing through (0, 0, 5), (1, 1, 4) and (2, -2, 1). Leave your answer in Ax + By + Cz = D form. [5]

$5C=1 \quad C=\frac{1}{5}$

$A+B+4C=1 \quad A+B=\frac{1}{5} \quad 2A+2B=\frac{2}{5} \quad B=-\frac{1}{10}$

$2A-2B+C=1 \quad 2A-2B=\frac{4}{5} \quad 4A=\frac{6}{5} \quad A=\frac{3}{10}$

$3x-y+2z=10$

3. Consider line L: $y=\frac{-2}{3}x+10 \longrightarrow \frac{2}{3}x+y-10=0$ $\quad y=\frac{3}{2}x+\frac{1}{2}$

$\frac{13}{6}x=\frac{9}{2}$

a) How far is line L from the origin? [2]

$$\frac{|-10|}{\sqrt{(\frac{2}{3})^2+1^2}}=\frac{10}{\sqrt{\frac{13}{9}}}=\frac{10}{\frac{\sqrt{13}}{3}}=\boxed{\frac{30}{\sqrt{13}}}$$

b) How far is line L from the point (1, 2) ? [2]

$$\frac{|\frac{2}{3}\cdot 1+1\cdot 2-10|}{\sqrt{(\frac{2}{3})^2+1^2}}=\frac{\frac{22}{3}}{\sqrt{\frac{13}{9}}}=\frac{\frac{22}{3}}{\frac{\sqrt{13}}{3}}=\boxed{\frac{22}{\sqrt{13}}}$$

c) Find both points on line y = 2x that are 5 units away from line L. [3]

$$\frac{|\frac{2}{3}x+2x-10|}{\sqrt{(\frac{2}{3})^2+1^2}}=5 \qquad |\frac{8}{3}x-10|=5(\frac{\sqrt{13}}{3})$$

① $\frac{8}{3}x-10=\frac{5\sqrt{13}}{3}$

$\frac{8}{3}x=\frac{5\sqrt{13}}{3}+10$

$8x=5\sqrt{13}+30$

$x=\frac{5\sqrt{13}+30}{8}$

② $-\frac{8}{3}x+10=\frac{5\sqrt{13}}{3}$

$\frac{8}{3}x=10-\frac{5\sqrt{13}}{3}$

$8x=30-5\sqrt{13}$

$x=\frac{30-5\sqrt{13}}{8}$

$\left(\frac{5\sqrt{13}+30}{8}, \frac{5\sqrt{13}+30}{4}\right)$ and $\left(\frac{30-5\sqrt{13}}{8}, \frac{30-5\sqrt{13}}{4}\right)$

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