$\frac{32}{32}$

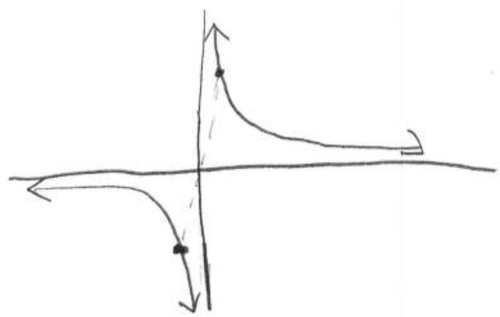
Limits Quiz Analysis 2013/14 I'm at my limit, and seriesly need a break

1. Consider the famous hyperbolic function $f(x)=\dfrac{1}{x}$.

a) Find the average rate of change over the interval x:[-1/3, 1/3].

$$\frac{f(\frac{1}{3})-f(-\frac{1}{3})}{\frac{2}{3}} = \boxed{9}$$

b) Sketch f(x) and a picture illustrating the geometric meaning of your answer from part "a". Explain in a sentence what your answer to part "a" represents.



answer to part a is the slope of the line going through $(-\frac{1}{3},-3)$ and $(\frac{1}{3},3)$

c) By looking at the graph does it appear that there is a point where the derivative (slope of the tangent line) is the same as your answer from "a"? Explain.

No, looking at the graph it looks like all derivatives of a specific point has a negative slope. 9 is a positive number.

d) Find the slope of the curve (instantaneous rate of change, or derivative) at any point x using algebra (not calculus). Answer in terms of x only.

$n \approx \infty$

$$\frac{f(x+\frac{1}{n})-f(x)}{\frac{1}{n}} = \left(\frac{1}{\frac{xn+1}{n}} - \frac{1}{x}\right)n = \left(\frac{n}{xn+1}-\frac{1}{x}\right)n$$

$$= \left(\frac{nx}{x^2n+x} - \frac{xn+1}{x^2n+x}\right)n = \frac{-n}{x^2n+x} \longrightarrow -\frac{1}{x^2}$$

2. Use finite differences to find the polynomial that will model the following data. Show the work that leads to your answer, and indicate when and how you used your TI.

x=1	2	3	4	5
y=4	-12	-64	-170	-348

4   -12   -64   -170   -348

16   52   106   178

36   54   72

18   18

cubic $= ax^3+bx^2+cx+$

$a+b+c+d=4$
$8a+4b+2c+d=-12$
$27a+9b+3c+d=-64$
$64a+16b+4c+d=-170$

plug in matrix in calculator

$$\left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 4 \\ 8 & 4 & 2 & 1 & -12 \\ 27 & 9 & 3 & 1 & -64 \\ 64 & 16 & 4 & 1 & -170 \end{array}\right]$$

Find rref of matrix: $\left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & -3 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 5 \\ 0 & 0 & 0 & 1 & 2 \end{array}\right] \rightarrow \boxed{f(x) = -3x^3+5x+2}$

3. Consider the sequence $t_n = \dfrac{-n}{2+3n}$, starting with n=1.

a) The limit of this sequence (as n goes to infinity) is $-\frac{1}{3}$

b) Is the sequence always increasing, always decreasing, or neither? Justify your answer using algebra.

decreasing

$t_n = \dfrac{-n}{2+3n}$

$t_{n+1} = \dfrac{-n-1}{2+3n+3}$

$\dfrac{-n-1}{3n+5} \le \dfrac{-n}{3n+2}$

$-3n^2-5n-2 \le -3n^2-5n \rightarrow -2 \le 0$ ✓

(question continued on next page)

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c) Using your answer from b (and some more logic) prove that the sequence converges. Write a conclusion statement showing why your proof is valid.

From part b we know the sequence is always decreasing. However, we know the limit is $-\frac{1}{3}$, and the equation cannot go below $-\frac{1}{3}$ or else it would have to increase to reach the limit. Therefore, as reaches ∞, the equation converges to $-\frac{1}{3}$.

d) Prove that your answer to part "a" is right by showing that after a certain term (M), the sequence will be within a neighborhood of radius E from your limit. Write a sentence at the end summarizing your logic.

$E = 0.01$

$$-\frac{1}{3} - E < -\frac{n}{2+3n} < -\frac{1}{3} + E$$

Case 1:

$$-\frac{1}{3} - E < -\frac{n}{2+3n}$$

$$-\frac{2}{3} - n - 2E - 3nE < -n$$

$$\frac{2}{3} + \cancel{n} + 2E + 3nE > \cancel{n}$$

$$-3nE < 2E + \frac{2}{3}$$

$$n > \frac{2E}{-3E} + \frac{\frac{2}{3}}{-3E}$$

$$n > -\frac{2}{3} - \frac{2}{9E}$$

n is positive so this always true

Case 2:

$$-\frac{n}{2+3n} < -\frac{1}{3} + E$$

$$\cancel{-n} < -\frac{2}{3} \cancel{-n} + 2E + 3nE$$

$$3nE > \frac{2}{3} - 2E$$

$$n > \frac{2}{9E} - \frac{2}{3}$$

$$n > \frac{2}{0.09} - \frac{2}{3}$$

$$n > \frac{194}{9} \approx 21.6$$

For any term of $M \geq 22$, if n is assigned M, it will be in the neighborhood.