Analysis Quest 2013/14 Interested Student [redacted] Per F

Calculators OK. You might use the following formulas during this quiz

$$FV = R\frac{(1+i)^n - 1}{i} \qquad PV = R\frac{1-(1+i)^{-n}}{i} \qquad i = \frac{r}{k}$$ 40/44

1. a) Your buddy Pierre comes to you in need of money. He proposes the following: If you loan him \$10,000 today, he will pay you back \$12,000 in 6 years. You know that you can get 4% interest in an investment account compounded monthly. You also know that Pierre is always true to his word. Should you take his offer? Defend your answer mathematically. [3]

$10000\left(1+\frac{0.04}{12}\right)^{72} = \$12707.42$

He will need to pay more than \$12000 in six years, so it is not a good offer.

b) Pierre instead proposes a payment plan where he borrows the \$10,000 and pays you back in \$150 payments every month for 6 years. You of course would immediately invest each \$150 in the bank (again at 4% monthly) Would this be a good deal for you? Justify your answer. [4]

$150\left[\frac{\left(1+\frac{0.04}{12}\right)^{72}-1}{\frac{0.04}{12}}\right] = \$12183.38$

This would not be a good deal, because in six years he needs to pay \$12707.42, but with this method, I only get \$12183.38.

c) What if Pierre can only afford \$100 monthly payments? How many monthly payments would he need to make in order for the deal to be fair to both you and Pierre? Again you are investing each payment in the bank. [4]

$100\left[\frac{\left(1+\frac{0.04}{12}\right)^{n}-1}{\frac{0.04}{12}}\right] = 10000\left(1+\frac{0.04}{12}\right)^{n}$

$\left(1+\frac{0.04}{12}\right)^{n} = \frac{1}{3}\left(1+\frac{0.04}{12}\right)^{n} + 1$

$\frac{2}{3}\left(1+\frac{0.04}{12}\right)^{n} = \times \frac{3}{2}$

$n = \log_{1+\frac{0.04}{12}} \frac{3}{2} = 121.84 \simeq$ 122 months

-0

2. A population of emus is shrinking by 7% every year. If there are 500 right now, how many emus will remain after 12 years? [3]

$500(1-0.07)^{12} = 209.298 \approx$ 209 emus

3. What interest rate would you need to triple your money in 20 years if your interest is compounded continuously? Assume no additional deposits to your account. Algebraic solution please. At the very end you may round to three decimal places. [3]

$x \cdot e^{20r} = 3x$

$e^{20r} = 3$

$20r = \ln 3$

$r = \frac{\ln 3}{20} =$ 5.49%

4. Consider a logistic equation in the form $y = \frac{A}{1+Be^{-kt}}$. The carrying capacity of the population is 100. After 1 year the population was 10, and after 4 years, the population was 20. Solve for k algebraically. No decimals in your final answer. [4]

$A = 100$

$\frac{100}{1+Be^{-k}} = 10$

$\frac{100}{1+Be^{-4k}} = 20$

$1+Be^{-k} = 10$

$1+Be^{-4k} = 5$

$Be^{-k} = 9 \quad B = \frac{9}{e^{-k}}$

$Be^{-4k} = 4 \quad B = \frac{4}{e^{-4k}}$

$\frac{9}{e^{-k}} = \frac{4}{e^{-4k}}$

$9 = \frac{4}{e^{-3k}}$

$9e^{-3k} = 4$

$e^{-3k} = \frac{4}{9}$

$-3k = \ln\left(\frac{4}{9}\right)$

$k = \frac{-\ln\left(\frac{4}{9}\right)}{3}$

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5. Consider the following logistic functions (or mutations of logistic functions)

a) $y=\frac{12}{1+5e^{-.3x}}$ b) $y=\frac{8}{1+4e^{2x}}+5$ c) $y=\frac{20}{6+e^{-2x}}$

Name the asymptote(s) for all three functions [2 pts each]

a) $y=0, y=12$ b) $y=5, y=13$ c) $y=0, y=20$ (-1) $y=\frac{10}{3}$

d) Which of the three functions above would be an example of "logistic decay"? Justify your answer. [2]

Function b would be an example of logistic decay, because as x gets bigger, y decreases.

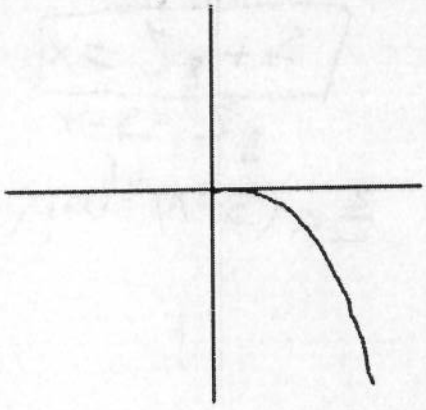
6. Find the point of inflection (as an ordered pair) for model "a" above. [3]

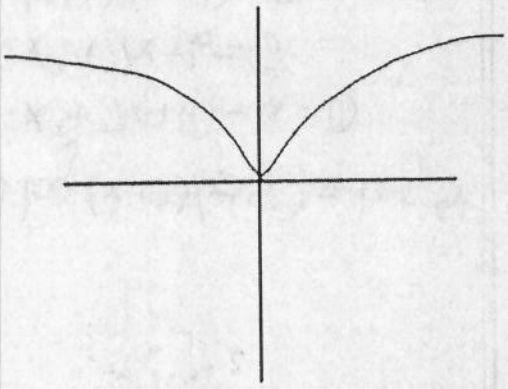
(0, 2) (-3) show work for partial credit. $\left(\frac{\ln 5}{0.3}, 6\right)$

$6=\frac{12}{1+5e^{-.3x}}$

$12=6+30e^{-.3x}$ $30e^{-.3x}=6$ $e^{-.3x}=\frac{1}{5}$ $-.3x=\ln\frac{1}{5}$ $.3x=\ln 5$

7. Below are the graphs of two power functions with rational exponents. Write a possible equation for each. There are many correct answers. [3 each]





a $y=-x^{\frac{5}{2}}$ b $y=x^{\frac{2}{3}}$

8. Solve the following for x: [3 each] (Fold over at dotted line and solve on the back)

a) $3\log_2(x-5)=7$ b) $\log(x+2)+\log(x+5)=2\log 2$

-4

(a) $\log_2(x-5) = \frac{7}{3}$

$x-5 = 2^{\frac{7}{3}}$

$\boxed{x = 2^{\frac{7}{3}} + 5}$

(b) $\log(x+2)(x+5) = \log 4$

$x^2+7x+10-4=0$

$x^2+7x+6=0$

$(x+6)(x+1)=0$

$\boxed{x=-1}$ ~~$x=-6$~~