[signature] loves math, and that's a Fact. Orial.
Period: [illegible]

Analysis H – Deggeller / Hahn
Ch1 Test – Alg through Problem Solving
CALCULATOR OK

75/75 wow.

Mystical Guess. Choose the best answer.

1. The statement, "Any given term $\binom{n}{k}$ is used to create the two terms below it, $\binom{n+1}{k}$ and $\binom{n+1}{k+1}$," is a proof of:

a) The hockey stick pattern in Pascal's Triangle

b) Each row is double the previous row in Pascal's Triangle

c) Finding triangular numbers in Pascal's Triangle

d) The middle term of the odd number triangle

e) The sum of each row in the odd number triangle adds up to n^3

6, 9, 12
9, 9, 9

2. A certain rectangular prism has edge lengths a, b, and c. Which of the following statements are true?

I. A cube whose edge length is the arithmetic mean of a, b, and c will have the same surface area as the prism.
II. A cube whose edge length is the geometric mean of a, b, and c will have the same volume as the prism.
III. A cube whose edge length is the arithmetic mean of a, b, and c will have the same total edge length as the prism.

a) I only b) II only c) III only d) I and II e) II and III

3. F_n is the nth Fibonacci number. Which of the following is NOT equivalent to F_n?

1 2 3 4 5 6 7 8 9 10
1, 1, 2, 3, 5, 8, 13, 21, 34, 55

a) $F_{n-1}+2F_{n-4}+F_{n-5}+F_{n-6}$

b) $F_{n-1}+F_{n-3}+F_{n-4}$

c) $3F_{n-3}+2F_{n-4}$

d) $8F_{n-5}+5F_{n-6}$

e) $F_{n-1}+F_{n-4}+2F_{n-5}+F_{n-6}$

Free Response: You may use a calculator, but you MUST show work! Correct answers with no work will receive no credit.

Evaluate each expression in terms of n.

4. $\sum_{k=1}^{n} 6k-3$ $3+9+15+21+27+\dots+6n-3$

$= \frac{3+6n-3}{2}\cdot n = 3n\cdot n = \boxed{3n^2}$

-0

5. $\prod_{k=1}^{n} 92$ $92\cdot 92\cdot 92\cdot 92\cdot \dots \cdot 92 = \boxed{92^n}$

Evaluate the expression in terms of n.

6. $\sum_{k=1}^{n} 3\left(\frac{2}{5}\right)^k$ $x = 3\left(\frac{2}{5}\right) + 3\left(\frac{2}{5}\right)^2 + 3\left(\frac{2}{5}\right)^3 + \cdots + 3\left(\frac{2}{5}\right)^n$

$\frac{5}{2}x = x + 3 - 3\left(\frac{2}{5}\right)^n$

$\frac{3}{2}x = 3 - 3\left(\frac{2}{5}\right)^n$

$x = \boxed{2 - 2\left(\frac{2}{5}\right)^n}$

✓

Write each as a single binomial coefficient.

7. $\binom{0}{213} + \binom{1}{213} + \binom{2}{213} + \ldots + \binom{n}{213}$

Using the Hockey Stick Theorem, this would equate to $\boxed{\binom{n+1}{214}}$ ✓

8. $\binom{6134}{5280} + \binom{6134}{5281} + \binom{6135}{5282}$

$\binom{6135}{5281}$

$\boxed{\binom{6136}{5282}}$ ✓

Evaluate. You may leave your answer in choose notation.

9. The coefficient of x^8y^{11} in the expansion of $\left(3x - \frac{7y}{10}\right)^{19}$

Binomial Theorem: $\binom{n}{r} a^r b^{n-r}$

$\boxed{\binom{19}{8} \cdot 3^8 \cdot \left(-\frac{7}{10}\right)^{11}}$ ✓

10. The coefficient of $x^3y^7z^5$ in the expansion of $(x+y+5z+w)^{15}$

$\binom{15}{3} \cdot \binom{12}{7} \binom{5}{5} \cdot 5^5 = \frac{15!}{3! \cdot 7! \cdot 5!} = \boxed{1126125000}$

✓

Simplify.

11. $\binom{-3}{75}$ $\frac{-3 \cdot -4 \cdot -5 \cdot -6 \cdot \ldots \cdot -77}{75!} = \frac{-1(\cancel{3 \cdot 4 \cdot 5 \ldots 77})}{\cancel{75!}} = \boxed{-2926}$

$\cancel{76} \cdot 77$

38

2

✓

0

1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89
144, 233, 377, 610, 987, 1597, 25

12. F_n is the nth Fibonacci number. Find a compact form for: $\sum_{k=1}^{n}\left(\frac{1}{F_{k+2}}-\frac{1}{F_k}\right)$

$\cancel{\frac{1}{2}}-\frac{1}{1}+\cancel{\frac{1}{3}}-\frac{1}{1}+\cancel{\frac{1}{5}}-\cancel{\frac{1}{2}}+\cancel{\frac{1}{8}}-\cancel{\frac{1}{3}}+\cancel{\frac{1}{13}}-\cancel{\frac{1}{5}}+\cdots+\frac{1}{F_{n+1}}-\cancel{\frac{1}{F_{n-1}}}+\frac{1}{F_{n+2}}-\cancel{\frac{1}{F_n}}$

$=\boxed{\frac{1}{F_{n+2}}+\frac{1}{F_{n+1}}-2}$

13. F_n is the nth Fibonacci number. Find a compact expression for: $\sum_{k=6}^{n}F_{2k}$

$F_{12}+F_{14}+F_{16}+F_{18}+\cdots+F_{2n-2}+F_{2n}$

$=\cancel{F_{13}}-F_{11}+\cancel{F_{15}}-\cancel{F_{13}}+\cancel{F_{17}}-\cancel{F_{15}}+\cancel{F_{19}}-\cancel{F_{17}}+\cdots+\cancel{F_{2n-1}}-\cancel{F_{2n-3}}+F_{2n+1}-\cancel{F_{2n-1}}$

$=F_{2n+1}-F_{11}=\boxed{F_{2n+1}-89}$

14. Given the geometric sequence 3, 6, 12… , which term has the value of 1,610,612,736?

Equation: $3\cdot 2^{n-1}$

$3\cdot 2^{n-1}=1610612736$

$2^{n-1}=536870912$

$n-1=29$

$\boxed{n=30}$

15. Find the 50th term of an arithmetic sequence where the third term is 12 and the eighth term is 2.

16 14 12 10 8 6 4 2 … equation: $16-2(n-1)$

$16-2(50-1)=\boxed{-82}$

16. Write 8 + 10 + 16 + 20 + 24 + … + 9,230 in an expression using sigma notation, where the series contains all the multiples of 8 or 10 (or both).

$\sum_{k=1}^{1153}8k+\sum_{k=1}^{923}10k-\sum_{k=1}^{230}40k=\boxed{8\sum_{k=1}^{1153}k+10\sum_{k=1}^{923}k-40\sum_{k=1}^{230}k}$

0

17. Prove by induction: $\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$

BASE CASE: $n=1$; $1^2 = \frac{1\cdot(1+1)(2\cdot1+1)}{6}$ ✓

ASSUME $n=a$ WORKS; $1^2+2^2+3^2+\cdots+a^2 = \frac{a(a+1)(2a+1)}{6}$

$n=a+1$; $\frac{a(a+1)(2a+1)}{6}+(a+1)^2 = \frac{a(a+1)(2a+1)}{6}+\frac{6(a+1)^2}{6} = \frac{(a+1)(a(2a+1)+6(a+1))}{6}$

$= \frac{(a+1)(2a^2+a+6a+6)}{6} = \frac{(a+1)(2a^2+7a+6)}{6} = \frac{(a+1)(a+2)(2a+3)}{6}$

✓ $= \frac{(a+1)((a+1)+1)(2(a+1)+1)}{6}$ ✓

18. Evaluate: $\sum_{y=8}^{25}\left[\sum_{x=1}^{10}(x+3y)\right]$

$= \sum_{y=8}^{25}(1+3y+2+3y+3+3y+4+3y+\cdots+10+3y)$

$= \sum_{y=8}^{25}(55+30y)$

$= 295+325+355+385+\cdots+805$ ✓

$= \frac{295+805}{2}\cdot 18$

$= \boxed{9900}$

-0