$\frac{64}{70}$

Calculus Chapters 1 & 2 [illegible signature] **is within Delta of summer break**
Analysis Test Deggeller/Hahn 2013-14 **Date** 5/2/14 **Period** F

Multiple Choice (3 pts each) **No Calculators on this page**

1. For what value of x does the following function have a vertical asymptote?

$$f(x)=\frac{(x+2)^2(x+3)}{x^2-4}$$

A) -2 B) -3 C) 0 D) 2 E) none of these

2. For what value of x does the following function have a removable discontinuity?

$$f(x)=\frac{(x+2)^2(x+3)}{x^2-4}$$

A) -2 B) -3 C) 0 D) 2 E) none of these

3. Evaluate the following limit

$$\lim_{x\to(-2)}\frac{(x+2)^2(x+3)}{x^2-4}$$

A) -4 B) -1/4 C) $\frac{1}{2}$ D) 0 E) Does Not Exist

4. Determine the value of k which would make the function $h(x)$ continuous

$$h(x)=\begin{cases}\dfrac{x^4-1}{x-1} & x\neq 1\\ x^2+k & x=1\end{cases}$$

$(x^2+1)(x+1)$ 4

x^2+k $1+k$

A) 4 B) 3 C) 2 D) 1 E) None of These

5. If $f(x)=\begin{cases}\dfrac{9}{x^2} & \text{if } x\le -3\\ 4+x & \text{if } x>-3\end{cases}$, then which of the following statements is **false**?

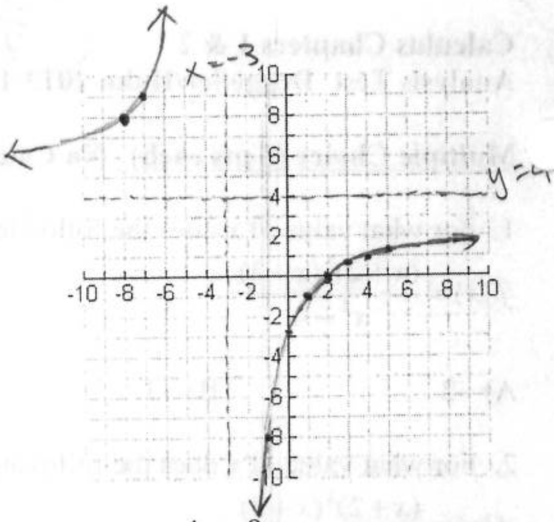
A) $\lim_{x\to -3^-} f(x)=1$ B) $\lim_{x\to -3^+} f(x)=1$ C) $\lim_{x\to -3} f(x)=1$ D) $\lim_{x\to -3} f(x)$ does not exist

6. $\lim_{x\to 4}\frac{x^3-64}{x^2-16}$ $\frac{\cancel{(x-4)}(x^2+4x+16)}{\cancel{(x-4)}(x+4)}$ $\frac{16+16+16}{8}=\frac{48}{8}=6$

A) 6 B) 16 C) 48 D) 0 E) Indeterminate

7a) Graph the function $f(x) = \dfrac{4x-8}{x+3}$ on the axes at the right.

Clearly label any asymptotes/holes. **[3 pts]**



b) Using some combination of deltas, epsilons, D's and E's, formally prove that $\lim_{x\to\infty} \dfrac{4x-8}{x+3} = 4$. Include a summary statement. [5]

$\dfrac{4x-8}{x+3} = 4 + \varepsilon$ (2)

$\cancel{4x} - 8 = \cancel{4x} + 12 + \varepsilon x + 3\varepsilon$

$3\varepsilon - 20 = \varepsilon x$

$x = 3 - \dfrac{20}{\varepsilon}$

If a number D is greater than $3 - \frac{20}{\varepsilon}$ for any $\varepsilon > 0$, then $f(x)$ is within ε of 4.

8. Find the following limits, or indicate that the limit does not exist (write "DNE"). **[2 pts each]**

$\lim_{x\to 3} g(x)$; for $g(x) = \dfrac{2x^2 - x - 15}{x-3}$ $\quad \dfrac{(2x+5)(x-3)}{(x-3)}$ 11

$\lim_{x\to 0} H(x)$; for $H(x) = \ln(\tan(x))$ $-\infty$ ok

$\lim_{x\to 4} G(x)$; for $G(x) = \dfrac{x+2}{x-4}$ DNE

$\lim_{x\to 4^-} R(x)$; for $R(x) = \sqrt{4-x} + x$ 4

$\lim_{x\to 4} R(x)$; for $R(x) = \sqrt{4-x} + x$ DNE

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Name [illegible]

Calculators allowed on this page only.

9. The rate at which population of Lemurs in Madagascar is changing can be modeled by the function $L(t) = 400te^{-t}$ where t is in ~~weeks~~ Months, and L is in lemurs/month.

a) Estimate L'(4.5) using your calculator (any method is fine just be clear about how you do it). Round to three decimal places. Include units. [3]

$$\frac{L(4.6)-L(4.5)}{4.6-4.5} = \frac{18.495-19.996}{0.1} = \boxed{-15.008 \ \frac{\text{lemurs}}{\text{months}^2}}$$

b) Use a trapezoidal sum to approximate the area under the curve over the interval t:[0,4] using 3 evenly spaced trapezoids. Clearly show your sum and avoid intermediate rounding. Include units. Clearly explain the meaning of this number. [5]

$$\frac{1}{2}\left(L(0)+2L\left(\tfrac{4}{3}\right)+2L\left(\tfrac{8}{3}\right)+L(4)\right) = 229.853 \rightarrow \boxed{229 \text{ lemurs}}$$

This is the total population change of lemurs from time 0 to time 4.

10. Construct a graph of a function that satisfies all of the given conditions: [6]

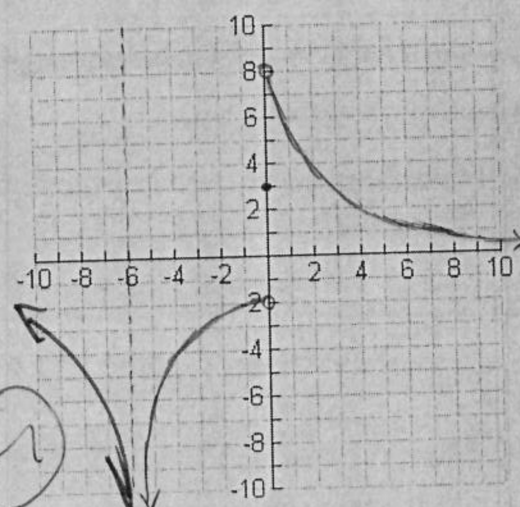
a) $\lim_{x \to 0^+} f(x) = 8$

b) $\lim_{x \to 0^-} f(x) = -2$

c) $f(0) = 3$

d) $\lim_{x \to -6} f(x) = -\infty$

e) The definite integral from x = 5 to x = 10 is equal to 5. (approximately)



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11. Fill in the blanks to complete the formal definition of a limit (use mathematical symbols and expressions when possible). [7]

"$\lim_{x \to c} f(x) =$ L if and only if for all values of δ, there exists some ε such that

if $0 < |x-c| < \delta$, then $|f(x)-L| < \varepsilon$."

-1

#12 and #13 are multiple choice. Choose the one best answer.

12. The function $f(x)=x^2-\sqrt{x}$ is continuous for x values [4,9]. The **intermediate value theorem** guarantees that: [3]

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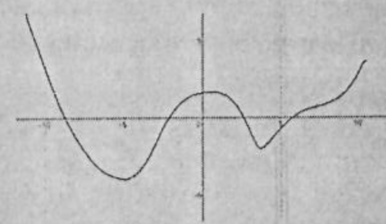
A) All x-values between 4 and 9 will output y values between f(4) and f(9).

B) All y-values between 4 and 9 can be attained

C) f(x) must have a local minimum or maximum between x=4 and x=9

D) There is some x-value between 4 and 9 such that f(x) = 50

E) There is no x-value between 4 and 9 such that f(x) = 100



13. Based on the graph to the right, which is the best approximation for $f'(-10)$? [3]

A) -2   B) -1   C) 0   D) 1   E) 2

14. Formally prove $\lim_{x\to4}\sqrt{x}+3=5$, by a delta/epsilon proof. Make sure you include a statement that summarizes your logic to complete the proof.

$5-\varepsilon<\sqrt{x}+3<5+\varepsilon$

$2-\varepsilon<\sqrt{x}<2+\varepsilon$

aller

$\varepsilon^2-4\varepsilon+4<x<\cancel{\varepsilon^2+4\varepsilon+4}$

$\delta=\varepsilon^2+4\varepsilon+4-4=\varepsilon^2+4\varepsilon$

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choose $\delta<\boxed{}$ -1

If x is within δ of 4, then $\sqrt{x+3}$ is within ε of 5.

15. Suppose that $\lim_{x\to\infty}\frac{rx^3+2x^2-17}{sx^3-30sx}=\sqrt{2},\ and\ r+s=2$, where r and s are constants. [3]

Find the exact value for s.

$\frac{r}{s}=\sqrt{2}$

$r=\sqrt{2}s$

$r+s=2$

$r-\sqrt{2}s=0$

$s+\sqrt{2}s=2$

$s=\frac{2}{1+\sqrt{2}}\left(\frac{1-\sqrt{2}}{1-\sqrt{2}}\right)$

$s=\frac{2-2\sqrt{2}}{-1}=\boxed{2\sqrt{2}-2}$

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