92/100

Analysis Matrices Test 2013/14 Decomposing student Jason [signature]
(ewwwwwwww…)

1. Circle **all** answers that are correct: The set of all natural numbers is the same size as

a) The set of all complex numbers

b) The set of all positive rational numbers

c) The set of all integers

d) The set of all points on a line

-1

2. Multiplying by the complex number $2cis\frac{5\pi}{6}$ has the same effect as multiplying by

the matrix: (give simplified answers) $\begin{bmatrix} -\sqrt{3} & -1 \\ 1 & -\sqrt{3} \end{bmatrix}$

$2cis\frac{5\pi}{6} = 2\left(-\frac{\sqrt{3}}{2} + \frac{1}{2}i\right)$
$= -\sqrt{3} + i$

3. The set of complex numbers a + bi (where a and b are real numbers not both zero) under the operation of multiplication is isomorphic to the set of matrices of the form:

$$\begin{bmatrix} a & -b \\ b & a \end{bmatrix}$$

4. Write a matrix that represents each of the following transformations. Give exact answers. Simplify sines and cosines when possible.

A) a rotation of $-45^\circ$

$\begin{bmatrix} \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \\ -\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \end{bmatrix}$

B) reflection over the y axis $x=0$

$\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$

C) reflects over the line y = - x

$\begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$

D) reflects over the line $\theta = 27^\circ$

$\begin{bmatrix} \cos 54^\circ & \sin 54^\circ \\ \sin 54^\circ & -\cos 54^\circ \end{bmatrix}$

5. State the matrix that does B and then A (from question 4 above):

$\begin{bmatrix} \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \\ -\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \end{bmatrix} \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} -\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \end{bmatrix}$

-1

6. Describe what each matrix does to the plane WITH A SINGLE TRANSFORMATION. Be specific with regards to magnitudes.

a) $\begin{bmatrix} \frac{-1}{2} & \frac{-\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{-1}{2} \end{bmatrix}$  b) $\begin{bmatrix} \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} & \frac{-\sqrt{2}}{2} \end{bmatrix}$  c) $\begin{bmatrix} 1 & 0 \\ 0 & 5 \end{bmatrix}$

rotates point 120° $\left(\frac{2\pi}{3}\right)$  reflect $\theta = \frac{45°}{2}$  stretch y by 5 times

d) $\begin{bmatrix} 1 & 0 \\ -3 & 1 \end{bmatrix}$  e) $\begin{bmatrix} 2/3 & 2/3 \\ 5/3 & 5/3 \end{bmatrix}$

shear y by -3 times  translates a point on the line $y = \frac{5}{2}x$

7. Decompose the matrix $\begin{bmatrix} A & B \\ C & D \end{bmatrix}$ as the product of four 2x2 matrices $R \cdot S \cdot T \cdot U$ in that order. (two shears and two stretches). Assume all entries are positive. Identify matrices R, S, T, and U below

$\begin{bmatrix} A & C \\ B & D \end{bmatrix}$  $\begin{bmatrix} 1 & \frac{C}{A} \\ B & D \end{bmatrix}$  $\begin{bmatrix} 1 & \frac{C}{A} \\ 0 & \frac{AD-BC}{A} \end{bmatrix}$

$$\begin{bmatrix} A & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ B & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & \frac{AD-BC}{A} \end{bmatrix}\begin{bmatrix} 1 & \frac{C}{A} \\ 0 & 1 \end{bmatrix} =$$

R= $\begin{bmatrix} A & 0 \\ 0 & 1 \end{bmatrix}$  S= $\begin{bmatrix} 1 & 0 \\ C & 1 \end{bmatrix}$  T= $\begin{bmatrix} 1 & 0 \\ 0 & \frac{AD-BC}{A} \end{bmatrix}$  U= $\begin{bmatrix} 1 & \frac{B}{A} \\ 0 & 1 \end{bmatrix}$

8. Use the geometry of complex numbers and Pascal's triangle to write a formula for $\cos 6\theta$.

$(\text{cis}\,\theta)^6 = \cos^6\theta + 6i\cos^5\theta\sin\theta - 15\cos^4\theta\sin^2\theta$
$- 20i\cos^3\theta\sin^3\theta + 15\cos^2\theta\sin^4\theta$
$+ 6i\cos\theta\sin^5\theta - \sin^6\theta$

1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1

$$\boxed{\cos 6\theta = \cos^6\theta - 15\cos^4\theta\sin^2\theta + 15\cos^2\theta\sin^4\theta - \sin^6\theta}$$

9. Consider the dihedral (reflection/rotation) group of a regular octagon.

a) How many elements are there in this group?

16

2 matrices

b) Write a rotation and reflection *matrix* that could generate this group.

$\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \end{bmatrix}$

c) Write two reflection matrices that could generate the same group.

$\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$

-2

10. a) Name a matrix that would shear the matrix $\begin{bmatrix} 3 & 5 \\ 2 & -4 \end{bmatrix}$ so that the first column vector would be on the **x axis**.

$\boxed{\begin{bmatrix} 1 & 0 \\ -\frac{2}{3} & 1 \end{bmatrix}}$

b) What matrix would ROTATE the above matrix so that the first column vector would be along the **y axis**? Simplify and write without trig functions for full credit.

$\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}\begin{bmatrix} 3 & 5 \\ 2 & -4 \end{bmatrix} = \cdots$

$\boxed{\begin{bmatrix} \frac{2\sqrt{13}}{13} & -\frac{3\sqrt{13}}{13} \\ \frac{3\sqrt{13}}{13} & \frac{2\sqrt{13}}{13} \end{bmatrix}}$

11. Write a 3 x 3 matrix that would

a) translate 5 units in the x and -4 in the y (in 2-d) and THEN dilate by 6.

$\cancel{\begin{bmatrix} 1 & 0 & 5 \\ 0 & 1 & -4 \\ 0 & 0 & 1 \end{bmatrix}}$

$\begin{bmatrix} 6 & 0 & 30 \\ 0 & 6 & -24 \\ 0 & 0 & \cancel{6} \end{bmatrix}$ -1

b) rotate the unit cube (represented by the 3 x 3 identity matrix) 90° from the first octant to the 5th octant (right below it)

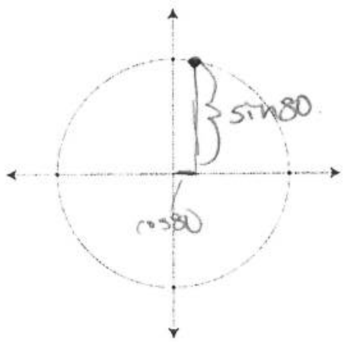
$\begin{bmatrix} 1 & 0 & 0 \\ 0 & \cancel{1} & \cancel{0} \\ 0 & \cancel{0} & \cancel{-1} \end{bmatrix}$ -2

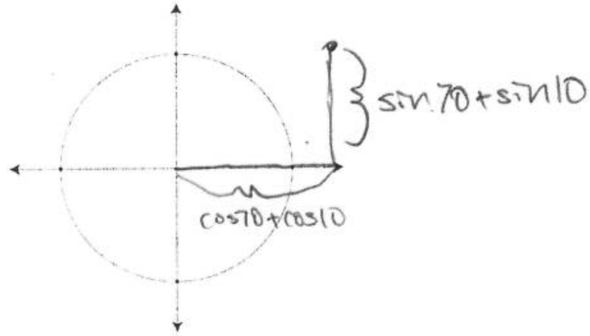
-5

12. a) Consider the two complex numbers $z_1 = cis10^\circ$ and $z_2 = cis70^\circ$.

Below accurately plot $z_1 \cdot z_2$ to the left and $z_1 + z_2$ to the right on the unit circles provided.

cos70+cos10+i(sin70+sin10)




b) $z_1$ generates a group under multiplication. What Geometric group is it isomorphic to?

36-gon rotation

c) Explain clearly why the group in part "b" above is actually a group (remember there are 4 requirements to be a group).

Has an identity ✓
Will come back to origin after full #s of rotation ✓
~~Will hit all points of geometric shape~~

-2

d) Would $z_1$ and $z_2$ generate a group under addition? Justify your answer.

No, because 0 is not part of the group and that is the identity.

-2