-15 $\frac{71}{86}$

Unit 4 Exam Analysis 2013/14 Student Lost in Space
3-D graphing and different types of coordinates

1. Identify the following 3-d surfaces by name: [3 each]

a) $2x + 3z = -16$ — a) plane

b) $y = 3z^2$ — b) parabolic cylinder

c) $2y^2 - 2x^2 + 5z = 2$ — hyperb. -2 — c) ~~elliptic~~ paraboloid

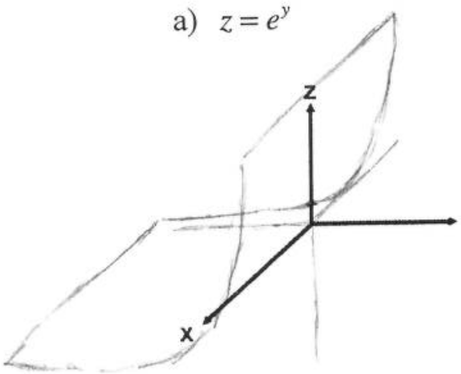
d) $(x-3)^2 + (y+5)^2 = z^2$ — d) elliptic cone

e) $x^2 + y^2 + 5z^2 = 22$ — e) ellipsoid

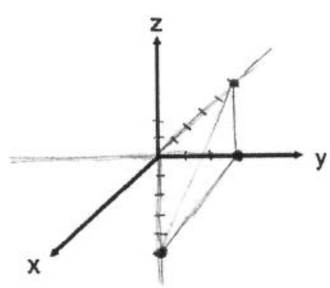
f) $2x^2 - y^2 + 3z^2 = 15$ — f) hyperboloid of 1 sheet

2. Sketch the following in 3 – d [4 each]

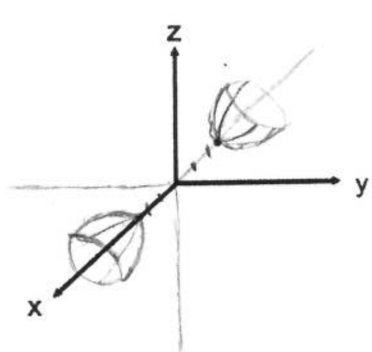
a) $z = e^y$



b) $-3x + 5y - 3z = 15$



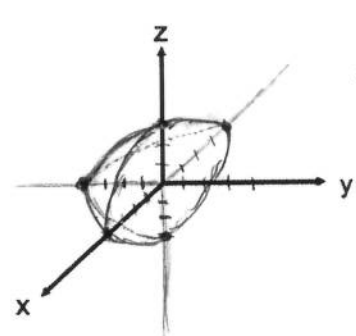
c) $\frac{z^2}{9} + \frac{y^2}{16} - \frac{x^2}{5} = -1$ $\quad \frac{x^2}{5} - \frac{y^2}{16} - \frac{z^2}{9} = 1$



d) $\frac{z^2}{9} + \frac{x^2}{16} - \frac{y}{4} = 1$

$\frac{y}{4} = \frac{x^2}{16} - 1$

$\frac{y}{4} = \frac{z^2}{9} - 1$



e) For "c" above state the domain (all possible x values) $-\infty < x \le -\sqrt{5},\ \sqrt{5} \le x < \infty$ — -2

3. Which one of the points below (in polar coordinates) does *not* map to the same point as $\left(4, -\frac{\pi}{3}\right)$? (circle one) [3 each for M.C.]

(a) $\left(-4, \frac{2\pi}{3}\right)$ (b) $\left(-4, -\frac{4\pi}{3}\right)$ (c) $\left(4, \frac{5\pi}{3}\right)$ (d) $\left(4, \frac{14\pi}{3}\right)$ ~~(e) $\left(-4, \frac{8\pi}{3}\right)$~~

-3

4. The graph of $r = -3\csc\theta$ can best be described as a… (circle one)

(a) horizontal line (b) vertical line (c) circle

(d) line with negative slope (e) line with positive slope

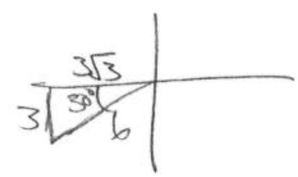
$r = -\frac{3}{\sin\theta}$ $y = -3$

5. Convert the point $\left(6, \frac{4\pi}{3}\right)$ from polar coordinates to rectangular coordinates. (circle one)

(a) $\left(-3\sqrt{2}, 3\sqrt{2}\right)$ (b) $\left(-3\sqrt{3}, 3\right)$ (c) $\left(-3, 3\sqrt{3}\right)$

~~(d) $\left(-3\sqrt{3}, -3\right)$~~ (e) $\left(-3, -3\sqrt{3}\right)$

-3



6. The polar function $r = 2\tan\theta\sec\theta$ is a parabola. Convert it to simplified rectangular form. [4]

$r = \frac{2\tan\theta}{\cos\theta}$

$r\cos\theta = 2\left(\frac{\sin\theta}{\cos\theta}\right)$

$r^2\cos^2\theta = 2r\sin\theta$

$\boxed{x^2 = 2y}$

7. Clearly describe the shape, size and orientation of the intersection of the two 3-d surfaces below. Be very specific. [4]

$x^2 + y^2 + z^2 = 120$ *and*

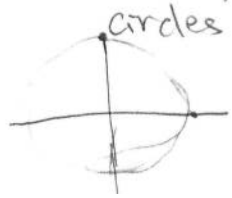
$x^2 + y^2 - z^2 = 100$

$2z^2 = 20$

$z^2 = 10$

$z = \sqrt{10}, -\sqrt{10}$

The shape of the intersections are two circles with centers of $(0,0,\sqrt{10})$ and $(0,0,-\sqrt{10})$, and both circles have a radius of ~~$2\sqrt{10}$~~. Both circles are also $\parallel$ ~~to~~ the x-y-plane.



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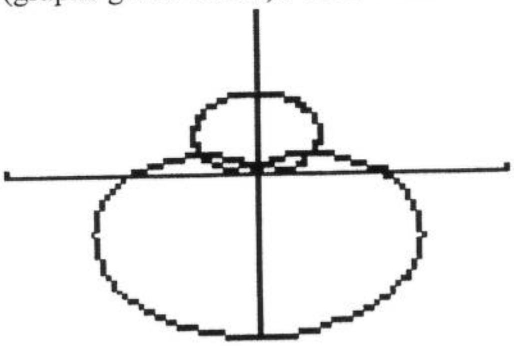
8. Find the geometric points of intersections of the two curves: $r = \sin\theta$ *and* $r = 1 - \sin\theta$ (graphs given below). Leave answers in the form (r, θ). [5]



$1 - \sin\theta = \sin\theta$

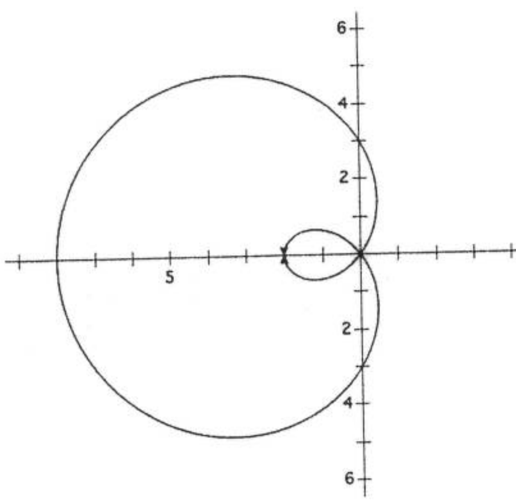
$2\sin\theta = 1$

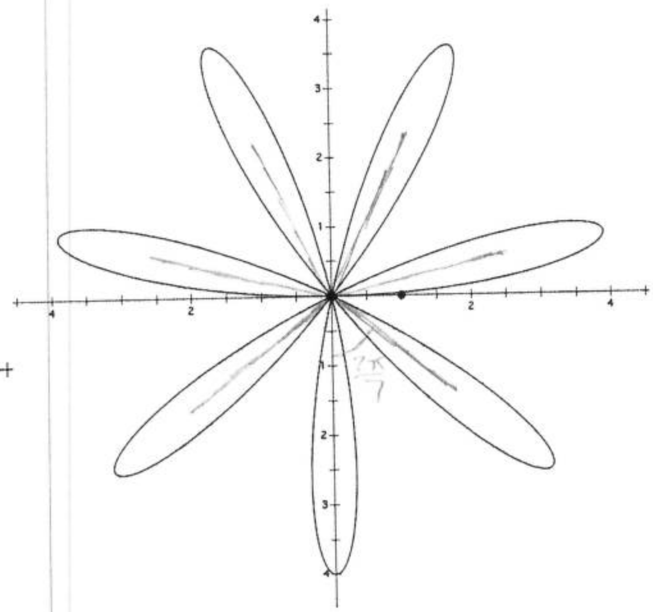
$\sin\theta = \frac{1}{2}$

$\theta = \frac{\pi}{6}, \frac{5\pi}{6}$

$\boxed{\left(\frac{1}{2}, \frac{\pi}{6}\right), \left(\frac{1}{2}, \frac{5\pi}{6}\right)}$ -1

9. Write an equation for the following two polar functions: [4 each]





r = $-3 - 5\cos\theta$

r= $4\sin 7\theta$

b) Along what angle is the first petal (past 0 degrees) in the rose above right? [2]

$\frac{2\pi}{7} - \left(\frac{\pi}{2} - \frac{2\pi}{7}\right) = \frac{4\pi}{7} - \frac{\pi}{2} = \boxed{\frac{\pi}{14}}$

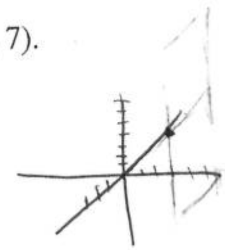
10. Write the equation of a plane with the following intercepts: (3, 0, 0) (0, 2, 0) and (0, 0, 5). [3]

$\boxed{10x + 15y + 6z = 30}$

-1

11. Name two planes that are parallel to the z axis that pass through the point (3, 5, 7). [3]

$x=3$ $\quad$ -3 $\quad$ $y=5$



Plane 1 ~~$7y+5z=35$~~ $\quad$ Plane 2 ~~$7x-3z=21$~~

12. The Cartesian coordinates for point P are (a, a, a) where "a" is a positive constant.

a) Write P in cylindrical coordinates (in terms of "a"). [3]

$r=\sqrt{a^2+a^2}=a\sqrt{2}$

$\theta=\tan^{-1}(1)=\frac{\pi}{4}$

$z=a$

$$\boxed{\left(a\sqrt{2}, \frac{\pi}{4}, a\right)}$$

b) Write P in spherical coordinates. (also in terms of "a") [3]

$\rho=\sqrt{a^2+a^2+a^2}=a\sqrt{3}$

$\theta=\tan^{-1}(1)=\frac{\pi}{4}$

$\phi=\tan^{-1}\frac{a\sqrt{2}}{a}=\tan^{-1}\sqrt{2}$

$$\boxed{\left(a\sqrt{3}, \frac{\pi}{4}, \tan^{-1}\sqrt{2}\right)}$$

13. a) Write the following spherical point Q= $(\rho,\theta,\phi)=(5,\frac{7\pi}{6},\frac{\pi}{6})$ as a rectangular point (x, y, z) [3]

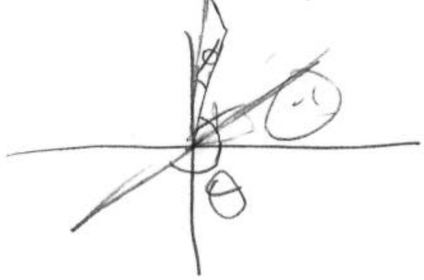
$x=5\sin\frac{\pi}{6}\cos\frac{7\pi}{6}=-\frac{5\sqrt{3}}{4}$

$y=5\cdot\sin\frac{\pi}{6}\cdot\sin\frac{7\pi}{6}=-\frac{5}{4}$

$z=\frac{5\sqrt{3}}{2}$

$$\boxed{\left(-\frac{5\sqrt{3}}{4}, -\frac{5}{4}, \frac{5\sqrt{3}}{2}\right)}$$

b) Sketch point Q, as accurately as possible, labeling (ρ,θ,ϕ) [3]



$\rho=5$ on diagram?

-1

$\theta=\frac{7\pi}{6}$

$\phi=\frac{\pi}{6}$

-5