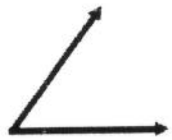
Name: [illegible]
Period: F Date: 12/12/13

$\frac{75}{80}$

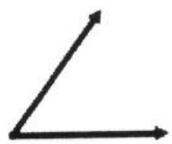
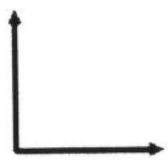
Analysis H – Deggeller/Hahn
Vectors and Parametric Quest
NO CALCULATOR

Multiple Choice. (4 pts each)

1. Which of the following pairs of vectors has the largest dot product?

a) 
b) 
c) 
d) 

1. Which of the following pairs of vectors has the cross product with the biggest magnitude?

a) 
b) 
c) 
d) 

3. One benefit of vectors is that they allow us to work in higher dimensions (4-D, 5-D, etc..), even though we may not be able to physically model them in our 3D world. Given vectors $\vec{a} = \langle 1,0,4,-2 \rangle$ and $\vec{b} = \langle 3,-7,2,1 \rangle$, find $\vec{a} \bullet \vec{b}$

$3+8-2=9$

a) 13 b) 11 c) 9 d) 7

4. *Using the items found in this classroom*: If vector **v** goes from the clock to the top of the door, and vector **u** goes from the middle of the floor to the speaker in the ceiling, which vector best represents **v** x **u**?

a) Smart Board to the back of the room
b) Smart Board to the door
c) Back of the room to the Smart Board
d) Door to the Smart Board
e) Speaker in ceiling to the middle of the floor

5. Which of the following could represent the direction cosines for a vector in 3-D space?

a) $\left\langle \frac{3}{\sqrt{14}}, \frac{1}{\sqrt{14}}, \frac{-2}{\sqrt{14}} \right\rangle$ b) $\left\langle \frac{-3}{\sqrt{14}}, \frac{5}{\sqrt{14}}, \frac{-2}{\sqrt{14}} \right\rangle$ c) $\left\langle \frac{2}{\sqrt{14}}, \frac{3}{\sqrt{14}}, \frac{4}{\sqrt{14}} \right\rangle$ d) $\left\langle \frac{2}{\sqrt{6}}, \frac{2}{\sqrt{6}}, \frac{2}{\sqrt{6}} \right\rangle$

6. Which of the following vectors is orthogonal to the vector $\langle 6,-2,4 \rangle$?

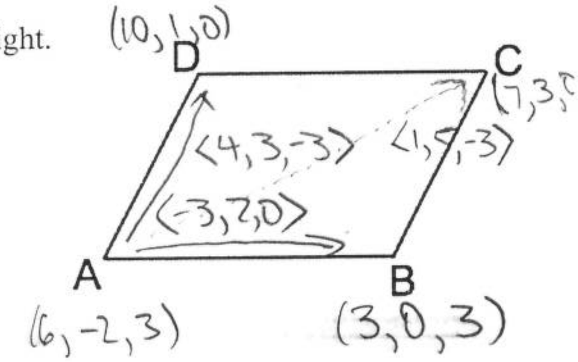
a) $\langle 2,3,-2 \rangle$ b) $\langle 1,1,1 \rangle$ c) $\langle 1,5,1 \rangle$ d) $\langle 2,7,-1 \rangle$

7. Answer "True" or "False" to each of the following statements. (2 pts each)

a) $\vec{u} \cdot \vec{v} = \vec{v} \cdot \vec{u}$	true	e) $(\vec{i} \times \vec{j}) \times \vec{i} = \vec{j}$	true
b) $\vec{u} \times \vec{v} = \vec{v} \times \vec{u}$	false	f) $(\vec{i} \times -\vec{j}) \times -\vec{i} = \vec{j}$	true
c) $\vec{u} \cdot (\vec{v} \cdot \vec{w}) = (\vec{u} \cdot \vec{v}) \cdot \vec{w}$	false	g) $(\vec{k} \times \vec{j}) \times \vec{j} = \vec{j}$	false
d) $\left\|\vec{u} \times \vec{v}\right\| = \left\|\vec{v} \times \vec{u}\right\|$	true	h) $\vec{k} \times (\vec{i} \times -\vec{j}) = \vec{j}$	false

Free Response.

8. ABCD is a parallelogram in space, as shown in the diagram on the right. Given point A (6, -2, 3), $\overrightarrow{AB} = \langle -3,2,0 \rangle$, and $\overrightarrow{AC} = \langle 1,5,-3 \rangle$, find:



a) point D (3 pts)

$(6+4, -2+3, 3-3) \rightarrow \boxed{(10,1,0)}$

b) Angle A (express your answer as an inverse cosine) (4 pts)

$\cos\theta = \dfrac{-12+6+0}{\sqrt{13}\sqrt{34}} = \dfrac{-6}{\sqrt{13}\sqrt{34}}$

$\boxed{\theta = \cos^{-1}\left(-\dfrac{6}{\sqrt{13}\sqrt{34}}\right)}$

c) $\overrightarrow{AB} \times \overrightarrow{AC}$ (4 pts)

$\begin{vmatrix} i & j & k \\ -3 & 2 & 0 \\ 1 & 5 & -3 \end{vmatrix} = i\begin{vmatrix} 2 & 0 \\ 5 & -3 \end{vmatrix} - j\begin{vmatrix} -3 & 0 \\ 1 & -3 \end{vmatrix} + k\begin{vmatrix} -3 & 2 \\ 1 & 5 \end{vmatrix} = -6i - 9j - 17k$

$\boxed{\langle -6, -9, -17 \rangle}$

c) The area of the parallelogram, using your answer from part (c) (3 pts) (leave your answer as a math expression - no need to compute/simplify).

$\begin{vmatrix} i & j & k \\ 4 & 3 & -3 \\ -3 & 2 & 0 \end{vmatrix} = 6i + 9j + 17k \rightarrow \boxed{\sqrt{6^2+9^2+17^2}}$

0

9. A quarterback throws a football, which then follows a path according to the following parametric equations (if the numbers look funny to you, it's because the quarterback is a giant with a weak arm).

$x = (24\sqrt{3})t$

$y = -16t^2 + 24t + 40$

$3t - 2t^2 = 0 \qquad t = \frac{3}{2} \qquad -16 \cdot \frac{4}{9} + 16 \qquad -\frac{16}{4} \; \frac{80}{9} \qquad -36 + 36 \qquad \frac{2}{3}$

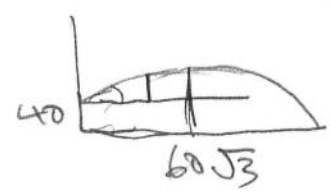
a) How far away from the quarterback will the ball hit the ground? (4 pts)

$-16t^2 + 24t = -40$

$2t^2 - 3t - 5 = 0 \Rightarrow (2t-5)(t+1)$

$t = \frac{3+\sqrt{9+40}}{4} = \frac{5}{2} \longrightarrow 2.5 \cdot 24\sqrt{3} = \boxed{60\sqrt{3}\,ft}$

b) How fast was the ball originally thrown (feet per second), and at what angle? (4 pts)



$\tan\theta = \frac{24}{24\sqrt{3}}$

$\theta = 30°$

$\frac{24}{\cos\theta} \rightarrow$ Initial velocity

-1

$\theta = \tan^{-1}\left(\frac{5\sqrt{3}}{9}\right)$

-2

10. Find the distance between the parallel planes $5x - 3y + z \; \cancel{-7=0} = 7$ and $5x - 3y + z \; \cancel{+3=0} = -3$. (5 pts)

$\langle 0,0,-3\rangle + t\langle 5,-3,1\rangle$

$\boxed{\text{distance} = \frac{2\sqrt{35}}{7}}$

$25t + 9t + t - 3 = 7 \quad 10$

$35t = 10$

$t = \frac{2}{7}$

$\frac{2}{7}\sqrt{25+9+1} = \frac{2}{7}\sqrt{35}$

11. Consider the planes $\quad -2x + y + z \; \cancel{-3=0} = 3 \quad$ and $\quad x - y + 2z \; \cancel{+4=0} = -4$

$\langle -2,1,1\rangle \qquad \langle 1,-1,2\rangle$

a) Find a vector equation of the line that is the intersection of the planes. (5 pts)

$-2x + y + z = 3 \qquad -4x + 2y + 2z = 6$

$x - y + 2z = -4 \qquad -5x + 3y = 10$

$-x + 3z = -1 \qquad y = \frac{5}{3}x + \frac{10}{3}$

$z = \frac{1}{3}x - \frac{1}{3}$

$x = t$

$y = \frac{1}{3}t$

$z = \frac{5}{3}t + \frac{10}{3}$

$\boxed{r = \langle 0, \frac{10}{3}, -\frac{1}{3}\rangle + t\langle 3,5,1\rangle}$

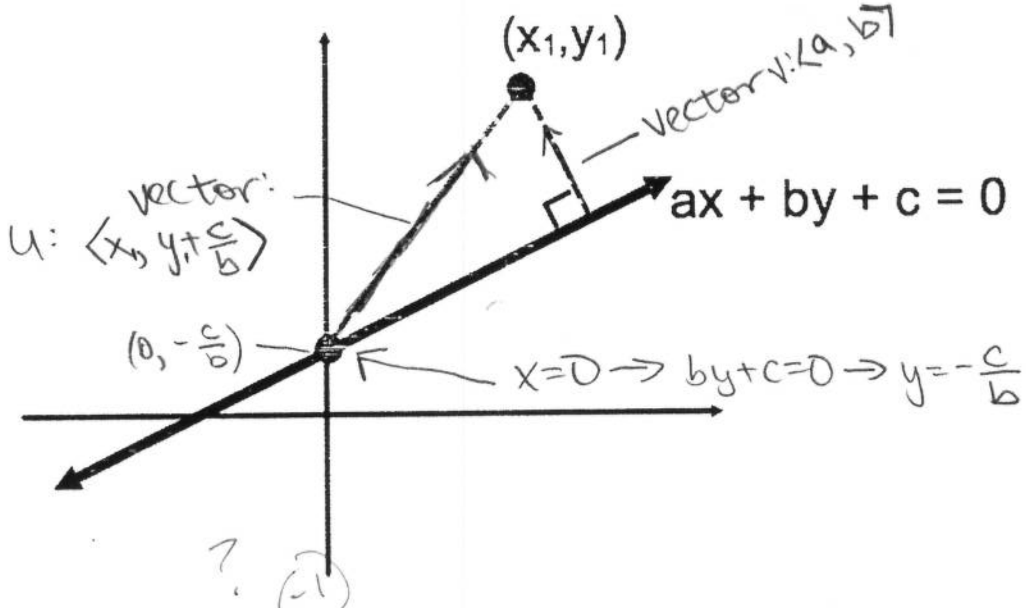
$\begin{vmatrix} i & j & k \\ -2 & 1 & 1 \\ 1 & -1 & 2 \end{vmatrix} = 3i + 5j + k$

b) Use your answer from part (a) to find a plane that is perpendicular to both planes. (4 pts)

$\boxed{3x + 5y + z = 0}$

-3

12. Use the following diagram and your knowledge of projections to derive the formula that calculates the distance between point (x_1, y_1) and the line $ax + by + c = 0$. Your answer will be graded on the clarity of your work, NOT on the accuracy of your final formula (duh.). Feel free to label the diagram as you see fit. (5 pts)



$$\left|\mathrm{proj}_v u\right| = \left|\frac{ax_1+by_1+c}{a^2+b^2}\langle a, b\rangle\right|$$

$$= \left|\left\langle a\left(\frac{ax_1+by_1+c}{a^2+b^2}\right), b\left(\frac{ax_1+by_1+c}{a^2+b^2}\right)\right\rangle\right|$$

$$= \sqrt{a^2\left(\frac{ax_1+by_1+c}{a^2+b^2}\right)^2 + b^2\left(\frac{ax_1+by_1+c}{a^2+b^2}\right)^2}$$

$$= \sqrt{\frac{\cancel{(a^2+b^2)}\,(ax_1+by_1+c)^2}{(a^2+b^2)^{\cancel{2}}}}$$

$$= \boxed{\frac{|ax_1+by_1+c|}{\sqrt{a^2+b^2}}}$$ (-1)

-2