Analysis H – Deggeller/Hahn   How many ways can Hannah Kim solve these problems?
Midterm 1 2017   **Non-graphing calculator ok**   Period: 6
ATPS Multiple Choice (2 pts each)

64/77

1. Given $\binom{156}{21}=\binom{155}{21}+\binom{w}{x}$ and $\binom{155}{20}=\binom{156}{20}-\binom{y}{z}$

a) x = 20 and z = 20   b) x = 22 and z = 19   c) x = 20 and z = 19   d) x = 21 and z = 20

e) x = 22 and z = 21

2. Given that F_n is the nth Fibonacci number, solve for x and y: $F_{500}=xF_{490}+yF_{491}$

a) x = 21, y = 34   b) x = 34, y = 55   c) x = 34, y = 21   d) x = 55, y = 21   e) x = 55, y = 34

−2

3. $F_{11}+F_{12}+F_{13}+.....+F_{400}=$

a) $F_{401}-F_{10}$ b) $F_{402}-F_{10}$ c) $F_{402}-F_{12}$ d) $F_{401}-F_{12}$ e) F_{403}

$F_{12}-F_{10}+F_{13}-F_{11}+F_{14}-F_{12}+\ldots+F_{401}-F_{399}$

$F_{401}+F_{400}-F_{10}-F_{11}=F_{402}-F_{12}$

4. $\sum_{k=0}^{n}\binom{m+k}{k}=$

a) $\binom{m+n+1}{n}$ b) $\binom{m+n}{n}$ c) $\binom{m+n}{n+1}$ d) $\binom{m+n+1}{n+1}$ e) $\binom{m+n}{n-1}$

−2

5. Use the factorial formula
$(n+1)!-n!=n!(n)$ to derive a compact expression for $3(3)!+4(4)!+......+100(100)!$

a) 100! – 3!   b) 101!   c) 101! – 3!   d) 100! – 4!   e) 101! + 4!

$4!-3!+5!-4!+6!-5!+\ldots+101!-100!$

6. What is the third term of an arithmetic sequence whose 20th term is 100 and 18th term is 94?

a) 43   b) 46   c) 49   d) 52   e) 55

−4

7. Which of the following is a term of the expansion $(2+\pi+x)^{100}$?

a) $\binom{100}{90}\binom{90}{10}2^{80}\pi^{10}x^{10}$ b) $\binom{100}{90}\binom{100}{10}2^{80}\pi^{10}x^{10}$ c) $\binom{90}{10}\binom{80}{10}2^{80}\pi^{10}x^{10}$

d) $\binom{100}{90}\binom{90}{10}(2\pi)^{10}x^{10}$ e) $\binom{100}{90}\binom{100}{10}2\pi^{10}x^{10}$

8. Which of the following is equivalent to $\binom{-3}{9}$?

a) 1/2 b) -1/2 c) 55 d) -55 e) none of the above

$\frac{n!}{(n-k)!\,k!} = \frac{-3!}{(-12)!\,9!} = \frac{\cancel{-3}\cdot\cancel{-4}\cdot\cancel{-5}\cdot\cancel{-6}\cdot\cancel{-7}\cdot\cancel{-8}\cdot\cancel{-9}\cdot\cancel{-10}\cdot -11}{2\cdot 1}$

Free Response:

9. Expand $\sum_{k=0}^{n+3} 12-3k =$ as a quadratic function in terms of n. Leave answer in the form an^2+bn+c [5]

$\sum_{k=0}^{n+3} 12-3k = 12-3(0) + 12-3(1) + 12-3(2) + \ldots + 12-3(n+3)$

$12 + 9 + 6 + \ldots + \cancel{12-3n-9}\ 3-3n$

$\frac{(15-3n)(n+3)}{2} = \frac{6n-3n^2+45}{2} = \boxed{\frac{-3n^2+6n+45}{2}}$ -1

$= \boxed{-3\left(\frac{n^2-2n-15}{2}\right)}$

10. For the series 3+ 3/2 + 3/4+.....

Find the sum of the first 13 terms (no need to simplify): [3]

$\frac{a+a(r^n)}{1-r} = \frac{3+3(\frac{1}{2})^{13}}{1-\frac{1}{2}} = \frac{3+3(\frac{1}{2^{13}})}{\frac{1}{2}} = (3(\frac{1}{2^{13}}+1))(2) = \boxed{6(\frac{1}{2^{13}}+1)}$

Find the sum of the infinite series, if it exists: [3] It does exist

$\frac{a}{1-r} = \frac{3}{1/2} = 6$

11. To prove that the formula for the sum of the first n cubes $=\frac{n^2(n+1)^2}{4}$ You would need to: [6 pts]

a) Show that 1^3 = $\frac{1^2(1+1)^2}{4} = \frac{1(2)^2}{4} = \frac{4}{4} = 1$

b) assume that the sum of the first k cubes = $\frac{k^2(k+1)^2}{4}$

c) Use step b to derive the fact that the sum of the first k+1 cubes = $\frac{(k+1)^2(k+2)^2}{4}$

(don't actually do the proof)

-1

Probability Multiple Choice (2 pts each)

12. $P(A) = 0.51$, $P(B) = 0.38$, and $P(A \cap B) = 0.15$, find $P(A' \cap B)$.

a) 0.23 b) 0.15 c) 0.11 d) 0.47 e) 0.26

13. $P(A) = 0.51$, $P(B) = 0.38$, and $P(A \cap B) = 0.15$, find $P(A \cup B)'$.

a) 0.23 b) 0.15 c) 0.11 d) 0.47 e) 0.26

$.51 + .38 - .15 \quad .89 - .15 = .74 \quad 100 - 74 = 26$

14. The events "Go to IKEA in the morning" and "Spend the evening making furniture" are...

a) Mutually Exclusive and Independent
b) Mutually Exclusive, but not Independent
c) Independent, but not Mutually Exclusive
d) Not Independent and not Mutually Exclusive
e) There is not enough information provided

−2

$\frac{10!}{5!\,2!\,2!}$

15. How many unique letter sequences can be created from the letters of GOOGOODOLL

a) $\frac{10 \cdot 9 \cdot 8 \cdot 7 \cdot 6}{4} = \frac{10!}{5!\,2!\,2!}$ b) $\frac{10!}{5 \cdot 2 \cdot 2}$ c) $10!$ d) $10! \cdot 2! \cdot 2! \cdot 5!$ e) $10 \cdot 9 \cdot 8 \cdot 7 \cdot 6$

For Questions 16-17: I go to Taco Bell once every 10 days (breakfast burritos, what whaaaaat!). The problem is, I have a weak stomach. And while I normally have a 10% chance of having an upset stomach, on days when I eat Taco Bell, that chance rises to 40%.

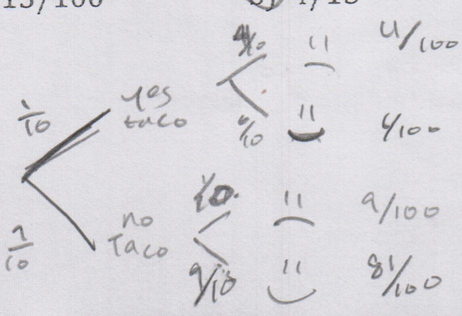
16. What is the probability that I have an upset stomach?

a) 10% b) 40% c) 25% d) 4% e) 13%

$9(.10) + 1(.40) = .9 + .4 = 1.3 \div 10 = .13$

17. What is the probability that I went to Taco Bell today, given that I have an upset stomach?

a) 13/100 b) 4/13 c) 4/100 d) 4/10 e) 1/10



/−2

For Problems 18-20: A certain school has 400 freshmen, 300 sophomores, 200 juniors, and 100 seniors (they have a terrible drop-out problem).

18. One day, the principal decides to give out a free Pearl Milk Tea to 30 lucky seniors (so nice!). How many different ways can this occur?

a) ${}_{100}C_{30}$ b) ${}_{100}P_{30}$ c) ${}_{1000}C_{30}$ d) 30! e) none of these

19. The same principal decides to give out Saturday School to 2 random students (so mean!) in the same grade. The principal will first randomly select a student from the school, and then randomly select another student from the same grade. What is the probability that Joey and his best friend (both juniors) will both get Saturday School?

a) $\frac{1}{1000}\cdot\frac{1}{199}$ b) $\frac{{}_{200}C_2}{{}_{1000}C_2}$ c) $\frac{1}{4}\cdot\frac{1}{199}$ d) $\frac{2}{{}_{200}C_2}$ e) $\frac{1}{1000}\cdot\frac{2}{199}$

−2

20. If you randomly select 6 students from the school, what is the probability that you will select 2 freshmen, 2 sophomores, and 2 juniors?

a) $\frac{\binom{200}{2}\binom{300}{2}\binom{400}{2}}{\binom{1000}{6}}$ b) $\frac{8}{1000}$ c) $\frac{\binom{4}{1}\binom{200}{2}\binom{4}{1}\binom{300}{2}\binom{4}{1}\binom{400}{2}}{\binom{1000}{6}}$

d) $\frac{\binom{4}{3}\binom{200}{2}\binom{300}{2}\binom{400}{2}}{\binom{1000}{6}}$ e) $1-\frac{\binom{100}{2}}{\binom{1000}{6}}$

Probability Free Response

21. Which of the following probabilities can be found using a **binomial distribution**? Check **ALL** that apply. [4 pts]

_√_ The probability that 3 out of 8 tosses of a coin will result in heads $\frac{1}{2}$, H or T

_√_ The probability that Susan, rolling a 6-sided die, will roll higher than Shannon, rolling a 10-sided die, in 7 out of the 9 times they each roll. $\left(\frac{872}{10!\,6!}\right)$, higher or lower

_X_ The probability of rolling at least two 3's and two 4's out of twelve rolls of a die. −2

(whatever the prob is) yes, at least 2 3's and 2 4's, or no, not at least

_X_ The probability of getting a full house in a poker hand.

unless it's rigged, same P(full house); full house or not full house

____ The probability that all 5 of your randomly-chosen group members passed the midterm

pass/fail ✓, but not same prob.

_√_ The probability that a student blindly guessing on a Scantron test will get at least 8 out of 10 multiple-choice questions correct (each with answer choices A-E).

$\frac{1}{5}$, correct or wrong

−4

22. Find the probability that Susan, rolling a 6-sided die, will roll higher than Shannon, rolling a 10-sided die, in 7 out of the 9 times they each roll [3]

10 9 8 7 6

$$\boxed{\binom{9}{7}\left(\frac{872}{6!\,10!}\right)^{7}\left(1-\frac{872}{6!\,10!}\right)^{2}}$$

6 || 5 — 1
5 || 4 — 1
4 || 3 — 1
3 || 2 — 1
2 || 1

$\frac{6!+5!+4!+3!+2!}{6!\,10!}$

$= \frac{2+6+24+120+720}{6!\,10!}$

$= \frac{872}{6!\,10!}$

−2

① 23. Gob is a travelling magician at a school, and goes around from classroom to classroom performing his magical illusions. Unfortunately, he's is not very good. For every illusion Gob performs, there is a 65% chance that the illusion will Slightly Underwhelm the Classroom of Kids (SUCK).

a) If Gob performs 14 illusions, what is the probability that at least 13 of them will SUCK? [3]

$$\binom{14}{13}(.65)^{13}(.35)^{1} + \binom{14}{14}(.65)^{14}$$

b) If Gob performs 14 illusions, what is the expected value for how many of them will SUCK? [1]

$$\cancel{14}^{\,7} \cdot \frac{\cancel{65}^{\,13}}{\cancel{100}\ \cancel{50}_{\,10}} = \frac{91}{10} = 9 \text{ illusions}$$

24. There are 3 identical boxes, A, B, and C. The chart below shows their contents.

Box A	Box B	Box C
3 gold coins	8 gold coins 2 silver coins	10 gold coins 10 silver coins

A: $\frac{1}{3}$, 1 G $= \frac{1}{3}$ G
B: $\frac{1}{3}$, $\frac{8}{10}$ G $= \frac{8}{30}$ G; $\frac{2}{10}$ S $= \frac{2}{30}$ S
C: $\frac{1}{3}$, $\frac{1}{2}$ G $- \frac{1}{6}$ G; $\frac{1}{2}$ S $- \frac{1}{6}$ S

You randomly select a box, then randomly select a coin from the box. [3 each]

a) P(the coin is silver)

$\frac{7}{30}$

b) P(silver coin | you didn't choose Box B)

$\frac{1}{4}$

c) P(you chose box C | the coin is gold)

$\frac{5}{23}$

$\frac{1}{3} \cdot 150$
$+ \frac{8}{30} \cdot 50$
$+ \frac{2}{30} \cdot 10$
$+ \frac{1}{6} \cdot 50$
$+ \frac{1}{6} \cdot 10$

50
$\frac{40}{3}$
$\frac{2}{3}$
$\frac{50}{6}$
$\frac{10}{6}$

$\frac{42}{3} = 14$

$\frac{60}{6} = 10$

74

d) If you choose a silver coin, you receive a $10 prize. If you choose a gold coin, you receive a $50 prize. If the gold coin happens to be from box A, you receive an extra $100 (on top of the $50 you also get for it being gold). What is the expected value of your winning?

$74

−2