Analysis H – Deggeller Hahn
Exam 2: Polar, 3D, vectors
NO CALCULATORS [94 points]

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Period: B

74/94 −20

Section 1: Polar and 3D graphing

Problems 1-6 are Multiple Choice: [3 each]

1. If a polar curve is symmetric over the line $\theta = \frac{\pi}{2}$ and contains the point (r,θ), name another point on the curve.

a) $(r,\pi-\theta)$ b) $(r,-\theta)$ c) $(-r,\theta)$ d) $(-r,\pi-\theta)$ e) $(r,\pi+\theta)$

2. The equation $r = 5 + k\cos\theta$ (where k > 0) will have an inner loop for what values of k?

a) k < 5 b) k > 5 c) 5 < k < 10 d) k = 5 e) k > 10

3. In the equation $r = 5 + k\cos\theta$ (where k > 0) what shape will be formed if k goes to infinity?

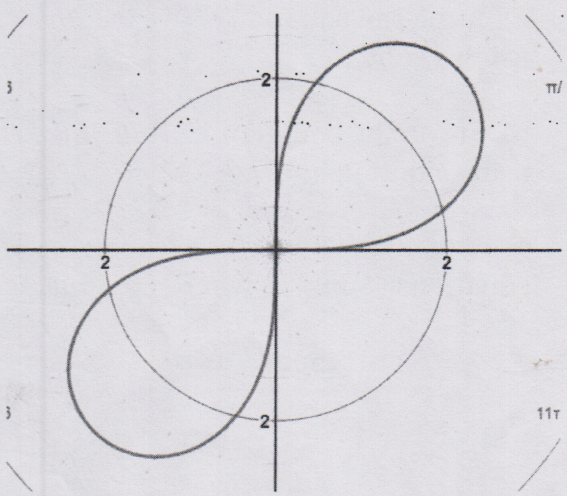
a) circle centered on the origin b) circle with a point on the origin

c) lemniscate d) line e) spiral

4. The curves of $r = 10$ and $r = 12\sin(4\theta)$ have ______ points of intersection.

a) 2 b) 4 c) 8 d) 12 e) 16

5. Which equation best models the graph on the right?



a) $r = 9\sin(2\theta)$ b) $r = 3\sin(2\theta)$

c) $r^2 = 9\sin(2\theta)$ d) $r^2 = 9\cos(2\theta)$

e) $r = 3\cos(2\theta)$

6. Convert the cylindrical point $(r,\theta,z) = \left(1, \frac{\pi}{2}, 1\right)$ into spherical coordinates (ρ,θ,ϕ).

a) $(0,0,1)$ b) $\left(\sqrt{2}, \frac{\pi}{2}, \frac{\pi}{4}\right)$ c) $\left(\sqrt{2}, \frac{\pi}{4}, \frac{\pi}{2}\right)$ d) $\left(\sqrt{2}, \frac{3\pi}{4}, \frac{\pi}{2}\right)$ e) $\left(\sqrt{2}, \frac{\pi}{2}, \frac{3\pi}{4}\right)$

−3

For problems 7-12, name the 3D surface that is produced by each equation (use proper mathematical vocabulary). [2 each]

7. $x^2 + y^2 = z^2$ double cone ✓

8. $x^2 + y^2 = z^2 + 2$ hyperboloid 1-sheet ✓

9. $x^2 - z^2 = y$ ~~parabolic cone~~ (-2)

10. $x - y^2 = z^2$ ~~saddle~~ (-2)

11. $x^2 + y^2 + z^2 = 8$ ellipsoid ✓

12. $x^2 = z^2$ X plane (2 planes) ✓

13. A certain parabolic cylinder that has a vertex at the point (0, 5, 0), hits the points (0, 0, 3) and (0, 0, -3), and is parallel to the x-axis.

a) Write the equation of the parabolic cylinder. [4]

$y^2 = z$

$(y-5)^2 = z$

$\frac{(y-5)^2}{25/3} = z$

$\frac{3(y-5)^2}{25} = z$ (-1)

b) When the parabolic cylinder intersects with the plane y + z = 3, what shapes are created in the intersection (be specific)? [2]

2 parallel lines ✓

c) Write the vector equation of **ONE** of the intersection shapes in part (b). [2]

$\frac{3(y-5)^2}{25} = 3 - y$

$3(y-5)^2 = 75 - 25y$

$3(y^2 - 10y + 25) = 75 - 25y$

$3y^2 - 30y + 75 = 75 - 25y$

$3y^2 = 5y$

$3y = 5$

$y = \frac{5}{3}$

$3 - \frac{5}{3} = z$

$\frac{4}{3} = z$

ok

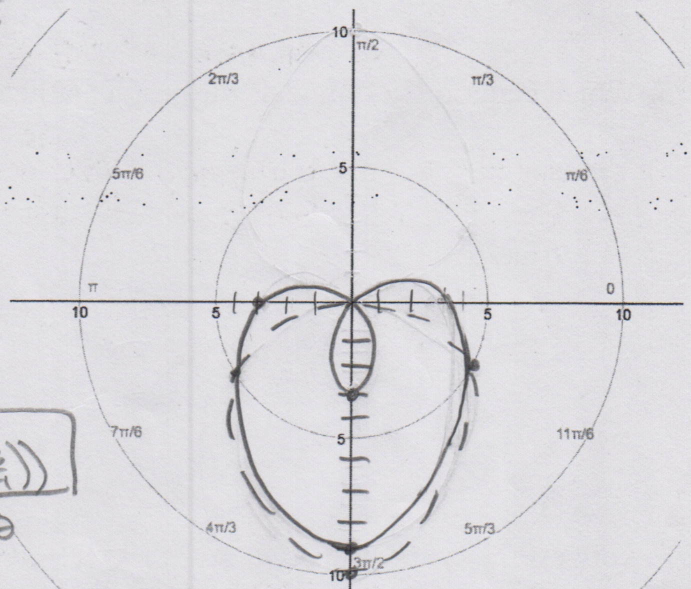
$\langle x, y, z \rangle = \langle 0, \frac{5}{3}, \frac{4}{3} \rangle + t\langle 1, 0, 0 \rangle$

14. On the graph to the right,

a) Graph the equation $r = 3 - 6\sin\theta$ using a solid line [3] ✓

b) Graph the equation $r = -10\sin\theta$ using a dotted line [2] ✓

c) Find all the points of intersection of the two equations. Simplify where possible. [4]



r	theta

(0, 0) ✓

$(7.5, \sin^{-1}(-\frac{3}{4}))$

$(7.5, \pi - \sin^{-1}(-\frac{3}{4}))$

$3 - 6\sin\theta = -10\sin\theta$

$3 = -4\sin\theta$

$-\frac{3}{4} = \sin\theta$

$r = -10 \cdot -\frac{3}{4} = 7\frac{1}{2}$

$3 - 6 \cdot -\frac{3}{4}$

$3 + \frac{18}{4} = 7\frac{1}{2}$

$(7\frac{1}{2}, \sin^{-1}(-\frac{3}{4}))$ ✓

$(7\frac{1}{2}, \pi - \sin^{-1}(\frac{3}{4}))$ ✓

$r = 3 - 6\sin(\frac{11\pi}{6})$

$= 3 - 6 \cdot \frac{1}{2}$

$= 3 + 3$

$= 6$

$(6, \frac{11\pi}{6})$

$(6, \frac{7\pi}{6})$

(-5

Section 2: Vectors and Parametrics

Problems 15-19 are Multiple Choice: [3 each]

15. Vectors **a** and **b** both lie in the xy plane and are in the first quadrant. If $a \cdot b = |a \times b|$, what can be concluded about a and b?

a) They are parallel

b) They are perpendicular

c) They make a 45 degree angle

d) They have the same magnitude

e) One has twice the magnitude of the other.

16. If the scalar projection $proj_b a$ is negative, what can be concluded about vectors a and b?

a) The angle between them is acute

b) The angle between them is obtuse

c) $|a| > |b|$

d) $|a| < |b|$

e) None of these must be true

17. Given vectors r and s, if $|r| > |s|$ then which of the following statements must be true:

a) $|proj_s r| > |s|$

b) $|proj_s r| = 0$

c) $|proj_s r| < |s|$

d) $|proj_s r| = |s|$

e) None of these must be true

18. Given points A, B and the origin O. Which of the following would be a method for finding the point 12 units from point A in the direction of vector $\overrightarrow{BA}$?

a) $\overrightarrow{OA} + \overrightarrow{BA} - <12,12>$

b) $\dfrac{\overrightarrow{BA}}{|\overrightarrow{BA}|} + 12\overrightarrow{OA}$

c) $12\dfrac{\overrightarrow{BA}}{|\overrightarrow{BA}|} + \overrightarrow{OA}$

d) $12\overrightarrow{BA} + \overrightarrow{OA}$

e) $12\overrightarrow{BA}$

19. The set of parametric equations $\begin{aligned} x &= 2\cos(t) + 2 \\ y &= 3\sin^2(t) - 5 \end{aligned}$ is a....

a) circle

b) ellipse

c) ray

d) partial parabola

e) hyperbola

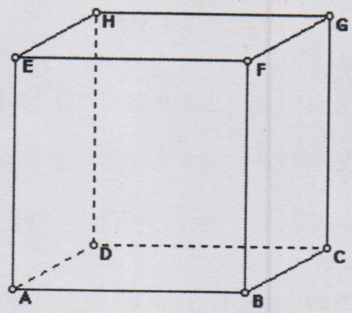
20. Answer the following True/False questions based on the unit cube below: [2 each]

a) $\overrightarrow{AE} \times \overrightarrow{AB} = \overrightarrow{AD}$ T

b) $\overrightarrow{GH} \times \overrightarrow{HE} = \overrightarrow{HD}$ F

c) $\overrightarrow{AB} \cdot \overrightarrow{FG} = 1$ F

d) $proj_{\overrightarrow{AB}} \overrightarrow{AG} = \overrightarrow{AB}$ T



21. Given two parallel planes $3x-2y+z=10$ and $3x-2y+z=-1$

(0, 0, -1) ← point on plane

a) Find the distance between the two planes, showing clearly that it is not 11. [3]

$\frac{|ax+by+cz+d|}{\sqrt{a^2+b^2+c^2}}$ $\frac{|3x-2y+z-10|}{\sqrt{3^2+2^2+1^2}} = \frac{|-1-10|}{\sqrt{9+4+1}} = \frac{11}{\sqrt{15}}$ units (-1)

dist. between plane 1 & point on plane 2

b) Find the equation of the line perpendicular to both planes that contains the point (2, 3, 5) [4]

$\langle 2,3,5\rangle + t\langle 3,-2,1\rangle$

$x=2+3t$
$y=3-2t$
$z=5+t$

$x+y-z=2+3t+3-2t-5-t$

~~$x+y-z=0$~~ (-2)

line = 1 variable
plane = 2 var.s

c) Find the rectangular form of a plane that is perpendicular to both planes and contains the points (2, 3, 5) and (0, 1, 0) [5]

$\langle x,y,z\rangle = \langle 0,1,0\rangle + t\langle 3,-2,1\rangle + l\langle 2,2,5\rangle$

$x=0+3t+2l$
$y=1-2t+2l$
$z=0+t+5l$

$x+y+z=1+2t+9l$

$x+y-z=1-l$

$-2y-x+z$

$-y=$ (-2)

-5

22. Is the point Q=(5, 3, 16) on the line $<x,y,z>=<1,2,2>+t<0,1,4>+s<-2,2,3>$? If so, justify by finding specific values of s and t that will generate Q. If not, justify. [4]

vector from O → Q

→ $\langle 5,3,16\rangle = \langle 1,2,2\rangle + t\langle 0,1,4\rangle + s\langle -2,2,3\rangle$

$\langle 4,1,14\rangle = t\langle 0,1,4\rangle + s\langle -2,2,3\rangle$

$\langle 4,-4,-6\rangle$

$\langle 4,1,14\rangle = t\langle 0,1,4\rangle + -2\langle -2,2,3\rangle$

$\langle 0,\not{3},\not{8}\rangle = t\langle 0,1,4\rangle$ (5, 20)

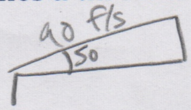
can't

smh u fucked up

(−1)

No. After turning Q into vector $\overline{OQ}$, I plugged it into the given equation. "s" must be −2 in order to get the desired "x", but there is no "t" that will satisfy what's left.

23. Dak Prescott is standing at midfield, 150 feet away from the goal line. He launches a throw from a height of 5 feet at a velocity of 90 feet per second at an angle of 50 degrees.



How far (horizontally) from the goal line will the ball be after t seconds? [3]

d = [illegible] (cos50)(90 ft/sec)

$150 - 90(\cos 50)t$ ft

24. Solve for the variables: $\langle 3,x,y\rangle\times\langle 4,9,2\rangle=\langle -15,-2,39\rangle$ [5]

$$\begin{vmatrix} i & j & k \\ 3 & x & y \\ 4 & 9 & 2 \end{vmatrix} = (2x-9y)i - (6-4y)j + (27-4x)k$$

$$\langle 2x-9y, -6+4y, 27-4x\rangle = \langle -15,-2,39\rangle$$

$-6+18$

y = 1

x = −3

−1