

$$\cos \theta = \frac{\vec{u} \cdot \vec{v}}{|\vec{u}| |\vec{v}|}$$

$$\text{proj}_{\vec{v}} \vec{u} = \frac{\vec{u} \cdot \vec{v}}{|\vec{v}|^2} \cdot \vec{v}$$

1. Given $\vec{u} = \langle 1, 4 \rangle$ and $\vec{v} = \langle -6, -1 \rangle$, find the following [2 points each]: Round to 3 decimal places when necessary and box your answers!!

a) $|\vec{v}| = \boxed{\sqrt{37}}$

b) The direction angle for \vec{v}
(between 0 and 360 degrees)

$$\tan^{-1} \left(\frac{-1}{-6} \right)$$

$$\boxed{189.462^\circ}$$

c) $2\vec{u} + \vec{v}$
 $2\langle 1, 4 \rangle + \langle -6, -1 \rangle$
 $\langle 2, 8 \rangle + \langle -6, -1 \rangle$
 $\boxed{\langle -4, 7 \rangle}$

d) $\vec{u} \cdot \vec{v}$
 $\langle 1, 4 \rangle \cdot \langle -6, -1 \rangle$
 $-6 - 4$
 $\boxed{-10}$

e) the angle between \vec{u} and \vec{v}
(in degrees)
 $\cos \theta = \frac{-10}{\sqrt{37} \sqrt{17}}$

$$\boxed{113.499^\circ}$$

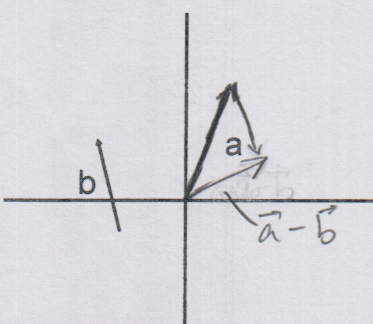
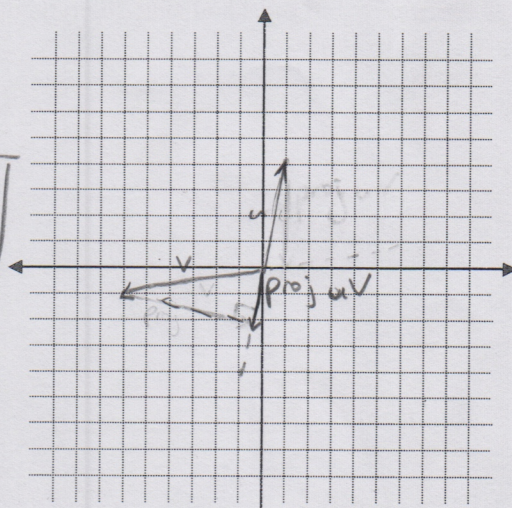
f) scalar $\text{proj}_{\vec{u}} \vec{v}$

$$\boxed{\frac{-10}{\sqrt{17}}}$$

g) vector $\text{proj}_{\vec{u}} \vec{v}$

$$\frac{-10}{17} \cdot \langle 1, 4 \rangle = \boxed{\langle -\frac{10}{17}, -\frac{40}{17} \rangle}$$

h) sketch \vec{u} , \vec{v} , and vector $\text{proj}_{\vec{u}} \vec{v}$ on the axes, to the right and label each



3. $\langle x_1, y_1 \rangle \cdot \langle x_2, y_2 \rangle$
 $x_1 x_2 + y_1 y_2$

see below $\rightarrow \langle 2x_1, 2y_1 \rangle \cdot \langle 2x_2, 2y_2 \rangle$
 $4x_1 x_2 + 4y_1 y_2$
 $4(x_1 x_2 + y_1 y_2)$

equal!
statement is false.

2. On the axes above left sketch and label the vector $\vec{a} - \vec{b}$. [2]

3. Prove or disprove the following statement: "If you double the magnitude of two vectors then you will double their dot product". Start your proof by introducing two generic vectors. [3]

doubling vector $\vec{v} = \langle x_1, y_1 \rangle$ $|\vec{v}| = \sqrt{x_1^2 + y_1^2}$
doubles magnitude $2\vec{v} = \langle 2x_1, 2y_1 \rangle$ $|2\vec{v}| = \sqrt{4x_1^2 + 4y_1^2} = 2\sqrt{x_1^2 + y_1^2}$
proof: doubled!

$$x = a - \sin t$$

$$\sin t = a - x$$

$$y = b + 2 \sin t$$

$$2 \sin t = y - b$$

$$\sin t = \frac{y - b}{2}$$

4. The following pair of parametric equations represents a line segment. [4]

$x = a - \sin t$ $y = b + 2 \sin t$ for $t : (-\infty, \infty)$ where a , and b are constants.

a) Write the line in rectangular (x, y) form, in terms of a and b .

$$a - x = \frac{y - b}{2}$$

$$2a - 2x = y - b$$

$$y = b + 2a - 2x$$

b) Name either of the two endpoints of the line segment (in terms of a and b).

$$(a + \sin \infty, b - 2 \sin \infty) \Rightarrow (a + 1, b - 2)$$

$$(a - \sin \infty, b + 2 \sin \infty) \Rightarrow (a - 1, b + 2)$$

4. The following parametric relation is **not** a line segment. Convert the relation to rectangular form, in terms of a and b . Then state the type of graph. [4] $x = b - \sin t$ $y = a + \cos t$ for $t : (-\infty, \infty)$

Relation _____

type of graph _____

5. A soccer ball is kicked off the ground with initial velocity of 50 f/s at an angle of 30 degrees. Leonardo is standing 40 feet away and is 5 feet tall. Will the ball go over Leonardo's head? (he's not allowed to jump) If so, how far over his head does it go? Remember to round to three decimal places! Clearly show the equations you used to algebraically solve this problem. [5]