Analysis H – Deggeller / Hahn — Building a pattern for success: Hannah Kim

Unit 1, Quiz 1 – Algebra through Problem Solving — Period: 3

NO CALCULATORS

31/32

1. Examine the following pattern of numbers. The last row shown is the 4th row. The middle term of the nth (when n is an odd number) row can be found by the formula $M(n) = 2n^2 + 5$.

1 · 4

2 · 4   4   7

3 · 4   11   15

4   19   23   27

4 · 4   12   31   35   39   43

47   51   55   59   64   −1

a) Write in the 5th row of the triangle. [3]

b) What is the first term of the 8th row? Show how you arrived at your answer. [3]

$\frac{7 \cdot 8}{2} = 7 \cdot 4 = 28 \quad 28 \cdot 4 = 112 \quad 112 + 7 = 119$

328
$\times 4$
$\overline{1124}$

c) Find an expression for the first term of the nth row. [3]

$\underbrace{1^{st} \text{ term of } n^{th} \text{ row}}_{} = T_1 + \left(\frac{(n)(n-1)}{2}(4)\right)$

$T_n = 7 + 2n^2 - 2n$

$T_1 = 7$

$F_{25} = F_{24} + F_{23}$

$= F_{23} + F_{22} + F_{23}$

$= 2F_{22} + 2F_{21} + F_{22}$

$= 3F_{21} + 3F_{20} + 2F_{21}$

2. Fill in the blanks. [3 pts per problem]

a) $F_{25} = \underline{5}\,F_{21} + \underline{3}\,F_{20}$

b) $F_{232} = F_{233} - F_{\underline{231}}$

c) $F_{17} + 2F_{18} + F_{19} + F_{20} = F_{\underline{22}}$

F_{19} F_{20} F_{20}

F_{21} F_{22}

−1

2. Simplify each. Write your answer as a single term or binomial coefficient (choose number) [3 pts for (a) – (c), 5 pts for (d)]

a) $\binom{47}{4}+2\binom{47}{5}+\binom{47}{6}=$ $\binom{49}{6}$ ✓

47

48

49

b) $\binom{61}{61}+\binom{62}{61}+\binom{63}{61}+\ldots+\binom{77}{61}=$ $\binom{78}{62}$ ✓

c) $\binom{86}{0}-\binom{86}{1}+\binom{86}{2}-\binom{86}{3}+\ldots+\binom{86}{86}=$ 0 ✓

d) Use words, diagrams, and/or math expressions to explain the pattern found in (b) OR (c) above (How do you know that this pattern works?). First, indicate which pattern you're explaining.

This is an explanation for (CIRCLE ONE) **(b)** **(c)**

You know this works bc the sum of the odd terms in a row & the sum of the even terms in a row will always be the same, and $x-x=0$. The sum of all the odd terms in a row is just the sum of all the terms in the previous row, and the sum of all the even terms in a row is also just the sum of all the terms in the previous row. ✓

In this case,

1 5 10 10 5 1

1 6 15 20 15 6 1

$6+20+6=32=1+15+15+1$.

We know this will always be true bc of this:

1 5 10 10 5 1 → 6 20 6

0 1 5 10 10 5 1 → 1 15 15 1

This pattern holds true for any row, because the entire structure of Pascal's triangle is based on x y