2016-2017 Analysis Probability Quiz 1 — 28/30

Hannah Kim is probably going to ace this

Period: B

***Please leave your answer in terms of C, P, exponents, and factorials. No need to evaluate to a number.*

1. I'm trying to figure out a good way to display my collection of Funko Pop dolls. I own 5 different Star Wars dolls, 7 different X-men dolls, and 14 different Pokémon dolls.

$5+7+14 = 21+5 = 26$

a) How many ways can I arrange them in a line on a shelf?

$\boxed{26!}$

b) How many ways can I arrange them on a shelf, if I want to group them together (Star Wars together, X-men together, and Pokémon together)?

$\boxed{3!(5!\,7!\,14!)}$

c) How many ways can I arrange them in a circle on my spinning circular table?

$$\frac{26!}{26} = \boxed{25!}$$

d) How many ways can I arrange them in a circle on my spinning circular table if I want to group them together?

$$\frac{3!\,(5!\,7!\,14!)}{3} = \boxed{2!\,(5!\,7!\,14!)}$$

e) I randomly pick 4 of the dolls. What is the probability that all the dolls are from Pokémon?

$$\frac{\cancel{14}^{7}}{\cancel{26}_{1}} \cdot \frac{\cancel{13}^{1}}{25} \cdot \frac{\cancel{12}^{1}}{\cancel{24}_{2}} \cdot \frac{11}{23} = \boxed{\frac{7 \cdot 11}{25 \cdot 23 \cdot 2}} \quad \text{or} \quad \boxed{\frac{14!\,22!}{26!\,11!}}$$

same things

-0

2. If you randomly select a 5-digit number, what is the probability that your number will contain only odd digits?

$$\frac{5^5}{9 \cdot 10^4}$$

3. In how many ways can we select two distinct integers from the set {1, 2, 3, ..., 100} so that the sum of the two numbers is even? -1

$$\frac{(50 \cdot 49)}{100 \cdot 99} + \frac{(50 \cdot 49)}{100 \cdot 99} = \frac{2 \cdot 50 \cdot 49}{100 \cdot 99} = \frac{49}{99} \quad (-1)$$

4. Given the letters of the word PREMONITION (11 letters in the word, with 2 O's, 2 N's, and 2 I's)...

a) How many ways can I rearrange the letters to create a distinct sequence of letters?

$$\frac{11!}{2! \, 2! \, 2!}$$

b) How many of the ways from (a) have the letters PRE together (but not necessarily in order)?

$$\frac{9! \, (3!)}{2! \, 2! \, 2!}$$

c) How many of the ways from (a) will have the letters PRE next to each other **in order**, and have the M appear somewhere after the PRE?

inorder: $\frac{9!}{2! \, 2! \, 2!}$

A B C D E F G H I

A _ _ B _ _ _ _ _

8 7 6 3 4 3 2 1

After PRE: $8 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$

$7! \cdot 8 \cdot 8$

$\frac{8 \cdot 8 \cdot 7!}{2! \, 2! \, 2!} = \boxed{\frac{9!}{2! \, 2! \, 2! \, 2}}$

$1 \cdot 8 \cdot 7!$

$1_2 \cdot 7 \cdot 7!$

$1_3 \, 6 \cdot 7!$

$1_4 \cdot 5 \cdot 7!$

$1_5 \cdot 4 \cdot 7!$

$1_6 \cdot 3 \cdot 7!$

$1_2 \cdot 2 \cdot 7!$

$1_8 \, 1 \cdot 7!$

-2