Analysis Honors – Deggeller / Hahn
Unit 8: Limits Quiz 1
NO CALCULATORS [31 points]

$\frac{32}{32}$

Hannah Kim is in the 'hood.
Period: B

For Questions #1-7, answer True or False for each statement. [2 pts each]

1. If a sequence is bounded below and always decreasing, it must converge. T
2. If a sequence is bounded above and below, it must converge. F
3. If a sequence is everywhere increasing, it must diverge. F
4. If a sequence has an upper bound, then it has an infinite number of upper bounds. T
5. If every other term of a_n is everywhere decreasing, then a_n is everywhere decreasing. F   sin/cos?
6. If 3 is the least upper bound of a_n, then a_n converges to 3. F
7. If a sequence is sometimes decreasing and sometimes increasing, then it cannot have a limit. F

For Questions #8-10, find the limit of each sequence, or say "diverges" if the sequence diverges. [2 each]

8. $a_n = \frac{\sin^2 n}{8n}$

0

9. $b_n = \frac{b_{n-1}}{2} + 3;\ \ b_1 = 8$

3

10. $c_n = 12 + \left(-\frac{1}{2}\right)^n$

12

11. Given the sequence $d_n = \frac{3n+5}{n+1}$...

a) The limit of the sequence is 3. [2]

b) **Prove** your limit from part (a) using a neighborhood of radius 0.1. Include a conclusion statement. [4]

$\frac{3n+5}{n+1} \geq 2.9$

$3n+5 \geq \frac{29n+29}{10}$

$\overset{21}{\cancel{30n+50}} \geq \cancel{29n+29}$

$n \geq -21$

Always ↑ bc n is natural

$\frac{3n+5}{n+1} \leq 3.1$

$3n+5 \leq \frac{31n+31}{10}$

$\overset{19}{\cancel{30n+50}} \leq \cancel{31n+31}$

$19 \leq n$

↑ n greater than/equal to 19

Conc: d_n is within the neighborhood of radius 0.1 for 3 when $n \geq 19$, showing that the sequence has a limit that it approaches as n increases.

−0

12. Prove that the sequence $t_n = \frac{1}{n+1} - 2$ converges to a limit by showing that it is everywhere increasing or everywhere decreasing (choose one), and bounded above or bounded below (choose one). Include a conclusion statement. [5]

$t_n = \frac{1}{n+1} - 2$

↳ always decreasing →

↳ bounded below at -2

$$\frac{1}{n+1} - 2 \le \frac{1}{(n-1)+1} - 2$$

$$\frac{-1-2n}{n+1} \le \frac{1-2n}{n}$$

$$\cancel{-n}\cancel{-2n^2} \le \cancel{n}+1\cancel{-2n^2}\cancel{-2n}$$

$$0 \le 1$$ ✓

$$\frac{1}{n+1} - 2 \ge -2$$

$$\frac{1}{n+1} \ge 0$$

$$1 \ge 0$$ ✓

$$\frac{1-2n-2}{n+1} \ge -2$$

$$-1\cancel{-2n} \ge \cancel{-2n}-2$$

$$0 \ge -1$$

Sequence t_n is everywhere decreasing & bounded below. In order to be both e.d. & bb., the sequence must approach a lower limit (which is the same as the lower bound.), otherwise it would cross the lower bound.

-0