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1. For each of the following pairs of sets A and B, indicate which is bigger (has more elements, or a bigger cardinality) set A or Set B. If the two sets are the same size write "same". [2 each]

A	B	answer
i) All multiples of 7	All whole numbers	same
ii) Real numbers between 0 and 1	Points on the y-axis	same
iii) Complex Numbers	Negative Integers	A

2. Is the group below under the operation \$ be isomorphic to the rotation group of a square? Give two pieces of evidence supporting your answer. [5]

\$	A	B	C	D
A	C	D	B	A
B	D	C	A	B
C	B	A	D	C
D	A	B	C	D

yes. has 4 elements like rot. group of square, also has corresponding elements.

$$D = I \quad \& \quad \# \$ D \text{ or } D \$ \# = \#$$

$$C = r^2 \quad \& \quad C \$ C = D$$

$$A \$ B = r \quad \& \quad r^3 \quad \& \quad A \$ B \text{ or } B \$ A = I$$

3. How many elements are there in the..... [2 each]

a) 2-d rotation group of a regular 17-gon? 17

b) The reflection group of a 3-d right prism whose base is an octagon? 32



4. Given complex numbers $b = \frac{3\sqrt{2}}{2} + \frac{3\sqrt{2}}{2}i$, find (simplified in a+bi form): [5 total]

$$\begin{aligned} \text{a) } b \cdot \bar{b} &= \left(\frac{3\sqrt{2}}{2} + \frac{3\sqrt{2}}{2}i \right) \left(\frac{3\sqrt{2}}{2} - \frac{3\sqrt{2}}{2}i \right) \\ &= \frac{9 \cdot 2}{4} - \frac{9 \cdot 2}{4} \cdot -1 = \frac{9}{2} + \frac{9}{2} \\ &= 9 + 0i \end{aligned}$$

$$\begin{aligned} \text{b) } b^{13} &= \left(3 \operatorname{cis} \frac{\pi}{4} \right)^{13} \\ &= 3^{13} \operatorname{cis} \frac{13\pi}{4} = 3^{13} \operatorname{cis} \frac{5\pi}{4} \\ &= 3^{13} \left(-\frac{\sqrt{2}}{2} + i - \frac{\sqrt{2}}{2} \right) \\ &= -\frac{3^{13}\sqrt{2}}{2} - \frac{3^{13}\sqrt{2}}{2}i \end{aligned}$$

5. Consider the complex number $G = 1cis80^\circ$.

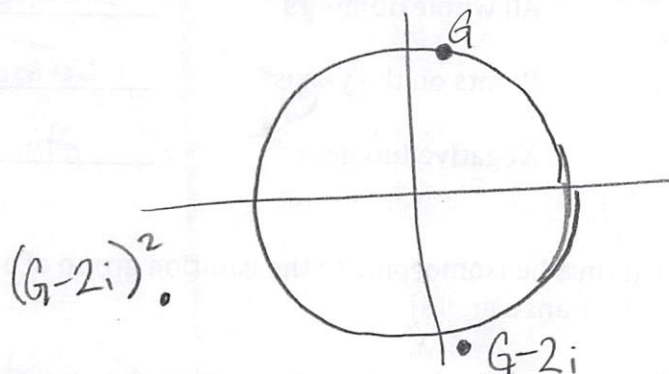


a) How many elements are in the group generated by G under multiplication? 9 [2]

b) Would $1cis40^\circ$ generate the same group? No [2]

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c) Below, draw a unit circle. Clearly label where the three points G , $G - 2i$ and $(G - 2i)^2$ would be. [5]



6. What transformation does the matrix below represent? Use specific mathematical language! [2]

$$\begin{bmatrix} 0 & 2 \\ 2 & 0 \end{bmatrix}$$

flip across $y=x$

dilation of 2

7. a) Find the image of point (a,b) under a reflection over the x axis and THEN a sheer in the y direction by k . [3]

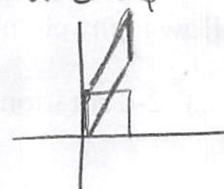
$$\begin{bmatrix} 1 & 0 \\ k & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ k & 1 \end{bmatrix} \begin{bmatrix} a \\ -b \end{bmatrix} = \begin{bmatrix} a \\ ak-b \end{bmatrix}$$

$$(a,b) \Rightarrow (a,-b)$$

$$\Downarrow$$

$$(a, ak-b)$$

vertical sheer



b) Prove that reversing the two transformations above would/would not (choose one) affect the final image of (a,b) . [2]

$$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ k & 1 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} \stackrel{\text{reflection}}{\neq} \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} a \\ ak+b \end{bmatrix} \stackrel{\text{sheer}}{=} \begin{bmatrix} a \\ -ak-b \end{bmatrix} \neq \begin{bmatrix} a \\ ak-b \end{bmatrix}$$

reversing transformations created a different image of (a,b)

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