ATPS Quiz 2 Hannah Kim is inducted into the Math Hall of Fame!
2016-17 Analysis [25 points] Period B

23/25

Calculators OK!! Please round all answers to 3 decimal places where appropriate.
3 points each problem (except for the induction proof, naturally)

1a) Consider an arithmetic sequence that begins with 3, and whose 30^{th} term is 500. Write in the second and third term.

500 − 3 = 497
497 ÷ 29 = 17.137931

3, 20.138 ✓, 37.276 ✓.....

b) Find the first term of a geometric sequence whose 2nd term is 5 and whose 50^{th} term is 500.

$500 = r^{50} \cdot m$ $5 = r^2 \cdot m$
$100 = r^{48}$

4.543 ✓

c) Consider an infinite geometric sequence that starts with 8, and whose sum is 50. Write in the second and third term.

$50 = \frac{8}{1-r}$

$50 - 50r = 8$
$-50r = -42$
$r = .84$

8, 6.72 ✓, 5.645 ✓,

d) The sum of the first n terms of an arithmetic series is 1550. The middle number is 10. How many terms are in the sequence?

$1550 = \frac{10(x)}{2}$

$3100 = 10x$
$= x$

310 ~~terms~~ −1

e) Three numbers have a geometric mean of 12 and an arithmetic mean of 30. One of them is 4. Find either of the other numbers.

$4 + b + c = 90$
$b + c = 86$

$\sqrt[3]{4bc} = 12$
$4bc = 1728$
$bc = 432$

$b = \frac{432}{c}$

$\frac{432}{c} + c = 86$

$432 + c^2 = 86c$

$c^2 - 86 + 432 =$

$\frac{43 \pm \sqrt{56.68}}{2}$ (−1)

$\frac{86 \pm \sqrt{7396 - 1728}}{2}$

$43 \pm$

−2

2. Solve for n: $\frac{(n-1)!(n+1)!}{n!(n-2)!}=17$

$(n+1)(n-1) = 17$

$n^2-1 = 17$

$n^2 = 18$

$\boxed{n = 3\sqrt{2}}$ ✓

3. Use mathematical induction to prove that the product of consecutive integers is even. In other words, that (n)(n+1) is a multiple of 2. Please clearly show all of the important steps of your proof and use words/math to justify any conclusion. [7]

$(n)(n+1) = 2m$ → shorthand for "multiple of 2"

SHOW $n=1$

$1(2) = 2m$ ✓

ASSUME $n=k$

$k(k+1) = 2m$

✓

PROVE $n=k+1$

$(k+1)(k+2)$

k^2+3k+2

$k^2+k+2k+2$

$\underbrace{k(k+1)}_{2m} + \underbrace{2k+2}_{2m} = 2m$

The sum of any even number + any other even number is even

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