

1. Identify each 3D shape by name, using proper mathematical vocabulary. [2 pts each]

a) $\frac{y^2}{4} = 5 - \frac{x^2}{7}$

~~ellipsoid~~ -2

b) $y^2 + x^2 = z - 4$

elliptic paraboloid ✓

c) $-z^2 + 4x^2 - y^2 = 0$

elliptic cone ✓

d) $2 + \frac{y^2}{8} = z + \frac{x^2}{8}$

saddle ✓

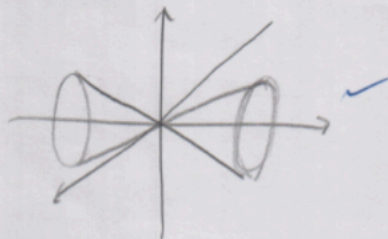
e) $\frac{x^2}{11} + z^2 = y^2 - 6$

hyperboloid of two sheets ✓

f) $y^2 + x^2 = -4$

not possible ✓

2. a) Sketch the shape defined by the equation $x^2 + z^2 = y^2$ [3 pts]



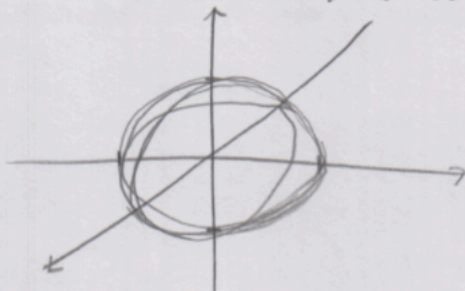
b) What shape(s) is/are formed by the intersection of the shape in part (a) with the plane $y = 8$? [1 pt]

a circle ✓

c) What shape(s) is/are formed by the intersection of the shape in part (a) with the plane $x = 3$? [1 pt]

a hyperbola ✓

d) Sketch the shape defined by the equation $x^2 + y^2 + z^2 = 30$ [3 pts]



It's an ellipsoid I swear
😊

e) What shape(s) is/are formed by the intersection of the shape in part (a) with the shape in part (d)? [1]

2 circles ✓

3. a) Write an equation of a Hyperboloid of 2 Sheets which opens up along the z-axis, has a vertex at $(0, 0, 6)$. [3 pts]

$x^2 - y^2 - z^2 = 1$

$(z-5)^2 - x^2 - y^2 = 1$

-1

b) Why are there infinite answers to question 3a above?

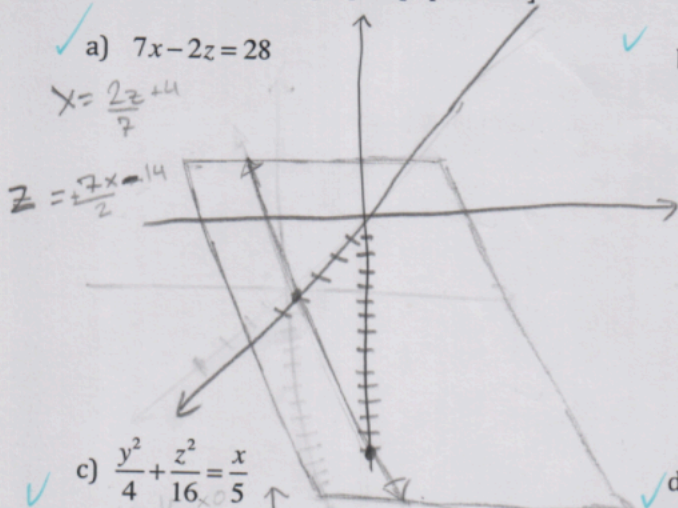
There are infinite answers because there are an infinite number of combinations of (x, y, z) that fit the equation $(z-5)^2 - x^2 - y^2 = 1$.

4. Sketch each 3D graph. [3 pts each]

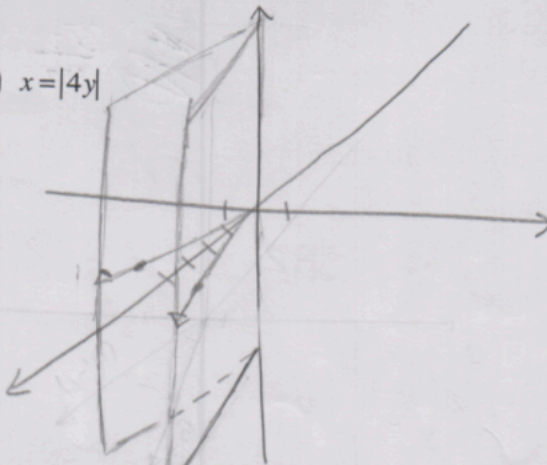
a) $7x - 2z = 28$

$x = \frac{2z + 28}{7}$

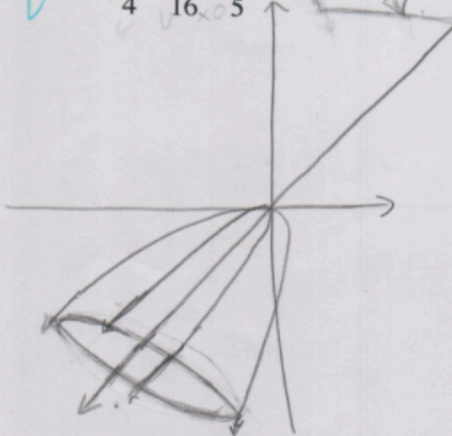
$z = \frac{7x - 28}{2}$



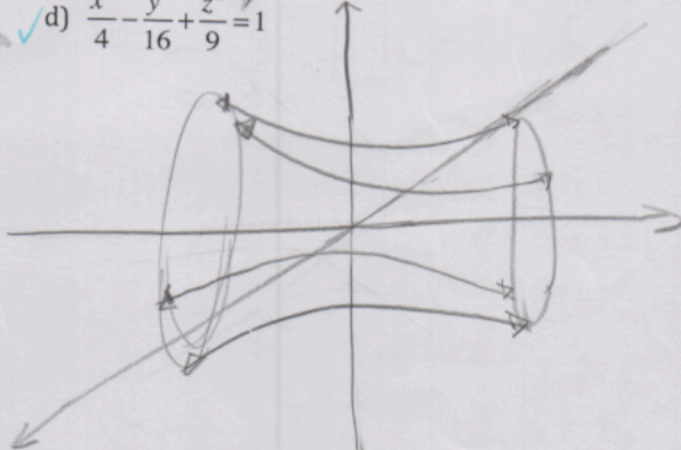
b) $x = |4y|$



c) $\frac{y^2}{4} + \frac{z^2}{16} = \frac{x}{5}$



d) $\frac{x^2}{4} - \frac{y^2}{16} + \frac{z^2}{9} = 1$



4. In **polar** coordinates, a certain conic section is defined by the equation $r = \frac{12}{2 - \cos \theta}$.

ellipse

$r = \frac{12}{1 - \frac{1}{2} \cos \theta} = \frac{\frac{1}{2} \cdot 12}{1 \pm \frac{1}{2} \cos \theta}$

(the general equations for polar conics are $r = \frac{ep}{1 \pm e \sin \theta}$ and $r = \frac{ep}{1 \pm e \cos \theta}$)

Make a sketch of the shape.

Accurately graph and label any vertices, foci, and directrices (multiples of each, if they exist).

[5 pts]

$\frac{\pi}{3}$ $\frac{6}{1 - \frac{1}{2}} = \frac{2 \cdot 6}{\frac{1}{2}} = 8$

$\frac{6}{1 - \frac{1}{2} \cdot \frac{\sqrt{3}}{2}} = \frac{24 \cdot 6}{24 \cdot \frac{4 - \sqrt{3}}{4}} = \frac{144}{4 - \sqrt{3}}$

$16 - 12 = \frac{4}{3}$

$(6 + \frac{4\sqrt{3}}{3}) = \sqrt{36 + \frac{16}{9}} + \sqrt{36 + \frac{20\sqrt{3}}{3}}$
 $= \sqrt{\frac{324}{9}} + \sqrt{\frac{324}{9}}$
 $= \frac{1}{3} \sqrt{324}$

