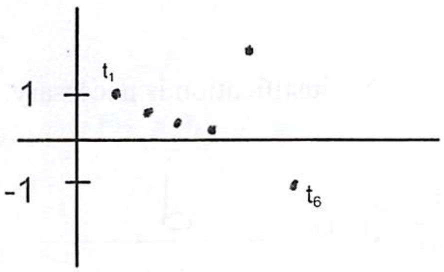
$\frac{38}{43}$

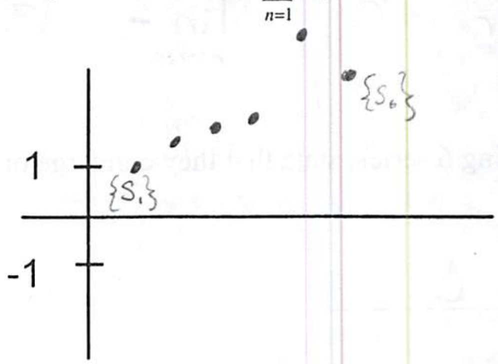
I see you studied Catherine's old quizzes

Analysis 2016-17 Student slowly converging on ~~summer~~ death Hannah Kim

Sequence and Series Quiz. No Calculators [43 pts]. Formulas written on board.

1. [3 pts] Sketched below is a sequence a_n (the 1[st] and 6[th] terms are labeled). Roughly sketch a picture of the first 6 terms of the sequence of partial sums for $\sum_{n=1}^{\infty} a_n$





2. Anwer True or False [2 each]

a) If a sequence $\{a_n\}$ converges to 1 then its corresponding series $\sum_{n=1}^{\infty} a_n$ also converges. F

b) If a sequence $\{a_n\}$ converges to 0 then the series $\sum_{n=1}^{\infty} a_n$ might diverge. F -2

c) If a sequence $\{a_n\}$ alternates, its terms approach zero, and its terms strictly decrease in absolute value then $\sum_{n=1}^{\infty} a_n$ must converge T

d) For sequence $\{a_n\}$ if the limit of $\frac{a_{n+1}}{a_n}$ approaches 2, then the corresponding series $\sum_{n=1}^{\infty} a_n$ might converge F

3. a) The sequence $t_n = \frac{5^n}{n!}$ begins to strictly decrease eventually. After which term is this true? (justify your answer algebraically) [3]

$$\frac{5^{n+1}}{(n+1)!} < \frac{5^n}{n!}$$

$$5^{\cancel{n+1}} < \cancel{5^n} \cdot (n+1)$$

$$5 < n+1$$

$$\boxed{4 < n}$$

$$\boxed{4}$$

? -1

b) Use the fact from part a (and other facts) to justify why $\sum_{n=1}^{\infty} \frac{5^n}{n!}(-1)^{n+1}$ converges or diverges.

[2]

AST:

1) alternating sign? ✓

2) strict decrease? ✓ (proof in 3a)

3) last term approaches 0? ✓

$$\frac{5 \cdot 5 \cdot 5 \cdot 5 \cdot 5 \cdot 5 \cdot 5 \cdot 5 \ldots 5 \cdot 5 \cdot 5 \ldots}{1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 \cdot 6 \cdot 7 \cdot 8 \cdot \ldots 625 \cdot 626 \cdot 627 \ldots}$$

← growth here is slower

← growth here is faster

eventually $n!$ overtakes 5^n and since growth rates differ the term approaches 0

-3

4. Given three sequences $\{r_n\}, \{s_n\}, \{t_n\}$ such that $\{r_n\} \le \{s_n\} \le \{t_n\}$ for all n.

What must be true about $\{r_n\}$ and $\{t_n\}$ to conclude that $\lim_{n\to\infty} s_n = 13$?

[2]

$\lim_{n\to\infty} r = 13 \qquad \lim_{n\to\infty} t_n = 13$

5. For the following 6 series, state that they converge or diverge. No justification is necessary. [2 pts each]

a) $\sum_{n=1}^{\infty} \frac{3}{5n^2}$ ___c___

b) $\sum_{n=1}^{\infty} \frac{13n-5}{3n+12}(-1)^n$ ___d___

c) $\sum_{n=1}^{\infty} \frac{n^3+3}{n^3+7}$ ___D___

d) $\sum_{n=1}^{\infty} \frac{7^n+1}{15^n}$ ___C___

e) $\sum_{n=1}^{\infty} \frac{(2n)!}{(3+n)^5\sqrt{n}}$ ___D___

f) $\sum_{n=1}^{\infty} \frac{1}{\sqrt[5]{n}}$ ___D___

6. Justify your answer to **5c** by using one of the tests that we learned in class. Include the name of the test in your answer. [2] $(n+1)^3 = n^3 + 3n^2 + 3n + 1$

$$\frac{\frac{(n+1)^3+3}{(n+1)^3+7}}{\frac{n^3+3}{n^3+7}} = \frac{(n^3+7)((n+1)^3+3)}{(n^3+3)((n+1)^3+7)} = \frac{n^3(n+1)^3+3n^3+7(n+1)^3+21}{n^3(n+1)^3+7n^3+3(n+1)^3+21} = \frac{n^6+3n^5+3n^4+n^3+3n^3+7n^3+21n^2+21n+7+21}{n^6+3n^5+3n^4+n^3+7n^3+3n^3+9n^2+9n+3+21}$$

$$= \frac{\boxed{n^6+3n^5+3n^4+11n^3}+21n^2+21n+28}{\boxed{n^6+3n^5+3n^4+11n^3}+9n^2+9n+24} > 1$$

same but $\rightarrow$ ~~$21n^2+21n+28 > 9n^2+9n+24$~~

diverging

~~$\boxed{\text{RATIO TEST}}$~~

-1

7. Justify your answer to **5f** by using one of the tests that we learned in class. Include the name of the test in your answer. [2]

$\boxed{\text{comparison}}$

$\frac{1}{\sqrt[5]{n}} > \frac{1}{n} \qquad \frac{1}{n}$ = diverging,

$n > \sqrt[5]{n}$, So $\frac{1}{\sqrt[5]{n}}$ must diverging

-1

8. Use the ratio test to prove whether $\sum_{n=1}^{\infty} \frac{10^n}{n4^{2n+1}}$ converges or diverges [4]

$$\lim_{n\to\infty} \frac{\frac{10^{n+1}}{(n+1)4^{2(n+1)+1}}}{\frac{10^n}{n4^{2n+1}}} = \frac{(10^{\cancel{n+1}})(n)(\cancel{4^{2n+1}})}{(\cancel{10^n})(n+1)4^{\cancel{2n+3}}} = \frac{10n}{4^2(n+1)} = \frac{10n}{16n+16} < 1$$

-1

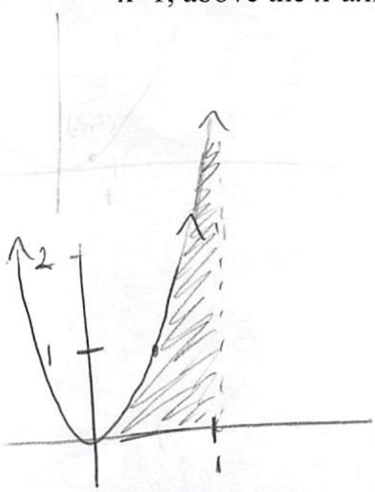
$\boxed{\text{Converges}}$

$10n < 16n+16$

$-6n < 16$

$n > -96$ ✓

9a) Use an infinite series to calculate the exact area under the curve $y = 4x^2$ between x = 0 and x=1, above the x-axis. Show all of your work. Include a picture [4]



$A = bh + bh + bh + bh + \ldots$

$A: \frac{1}{n}4\left(\frac{1}{n}\right)^2 + \left(\frac{1}{n}\right)4\left(\frac{2}{n}\right)^2 + \ldots + \left(\frac{1}{n}\right)4\left(\frac{n}{n}\right)^2$

$= \left(\frac{1}{n}\right)^3 4\left(1^2 + 2^2 + \ldots + (n)^2\right)$

$= \left(\frac{1}{n}\right)^3 (\cancel{4}^{2}) \frac{(n)(n+1)(2n+1)}{\cancel{6}\,3}$

$\lim_{n\to\infty} \frac{2\cancel{n}(n+1)(2n+1)}{3n^2} = \boxed{\frac{4}{3}}$

$= 16n^2$

b) How would your answer change if the function were $y = 4x^2 + 7$ instead? [1]

$\frac{4}{3} + 7 = \frac{4}{3} + \frac{21}{3} = \boxed{\frac{25}{3}}$

-1