Analysis Honors – Deggeller / Hahn

Hannah Kim won't get caught in a trap… rule.

Calc Quiz 1

Period: B

Calculator OK

-4 $\quad \frac{46}{50}$

For questions 1-8, refer to the function: $f(x) = x^3 - 2x + 3,$

1. What is the average rate of change of f(x) over the x interval [-1, 5]?

$$\frac{f(5) - f(-1)}{5 - -1} = \frac{(5^3 - 2(5) + 3) - ((-1)^3 - 2(-1) + 3)}{6} = \frac{114}{6} = \boxed{19}$$

2. What is the derivative of f(x) at x = 1? Find your answer by using the **Definition of a Derivative at a Point**.

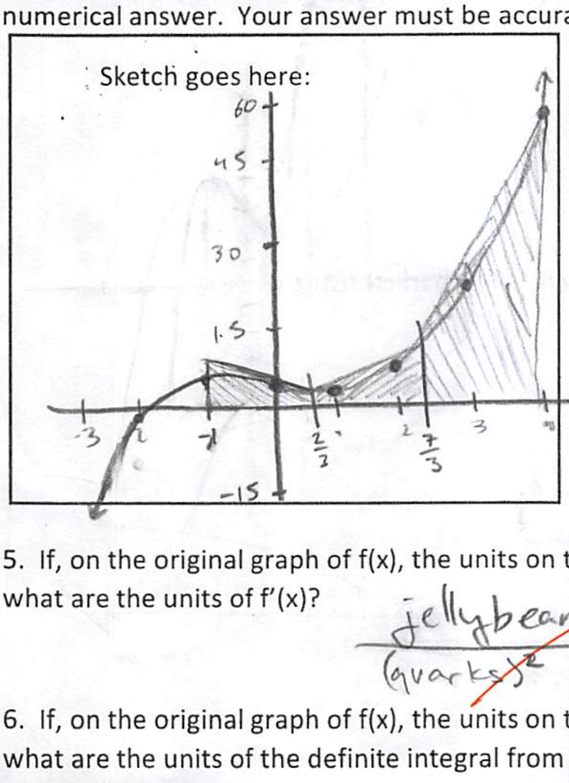
$$f'(1) = \lim_{h\to 0} \frac{f(1+h) - f(1)}{h} = \lim_{h\to 0} \frac{(h+1)^3 - 2(h+1) + 3 - 2}{h} = \lim_{h\to 0} \frac{h^3 + 3h^2 + 3h + \cancel{1} - 2h + \cancel{-2} + \cancel{3} \cancel{-2}}{h}$$

$$= \lim_{h\to 0} \frac{h^3 + 3h^2 + h}{h} = \lim_{h\to 0} \underbrace{h^2}_{0} + \underbrace{3h}_{0} + 1 = \boxed{1}$$

3. What is the equation of the line tangent to f(x) at x = 1?

$$(y - 2) = (x - 1)$$

4. Use 3 equally-spaced trapezoids to approximate the definite integral of f(x) over the x-interval [-1,4]. For full credit, include a labeled sketch of the trapezoids on the function and the set up of the trapezoidal summation, as well as your numerical answer. Your answer must be accurate to 3 decimal places.



$$\left(\frac{f(\frac{2}{3}) - f(-1)}{\cancel{\frac{2}{3} - -1}}\right) + \left(\frac{f(\frac{7}{3}) - f(\frac{2}{3})}{\cancel{\frac{7}{3} - \frac{2}{3}}}\right) + \left(\frac{f(4) - f(\frac{7}{3})}{\cancel{4 - \frac{7}{3}}}\right)$$

-2

$$= \frac{\frac{53}{27} - 4}{\frac{5}{3}} + \frac{\frac{298}{27} - \frac{53}{27}}{\frac{5}{3}} + \frac{59 - \frac{298}{27}}{\frac{5}{3}}$$

$$= \frac{59 - 4}{\frac{5}{3}} = \frac{55}{\frac{5}{3}} = \cancel{33}$$

wyd fam??

5. If, on the original graph of f(x), the units on the x-axis are "quarks", and the units on the y-axis are "jellybeans", then what are the units of f'(x)?

$$\frac{\text{jellybeans}}{\cancel{(\text{quarks})^2}} \quad -1 \quad \boxed{\frac{\text{jellybeans}}{\text{quarks}}}$$

6. If, on the original graph of f(x), the units on the x-axis are "quarks", and the units on the y-axis are "jellybeans", then what are the units of the definite integral from question #4?

jellybeans · quarks

-3

Yep, still refer to the function: $f(x) = x^3 - 2x + 3$,

7. What is $\lim_{x \to 2} f(x)$?

7

8. How close to 2 does the x-value need to be in order to ensure that f(x) is within 0.05 of your answer from #7? Show the thinking that leads to your answer.

$x^3 - 2x + 3 = 6.95$ $\quad$ $x^3 - 2x + 3 = 7.05$

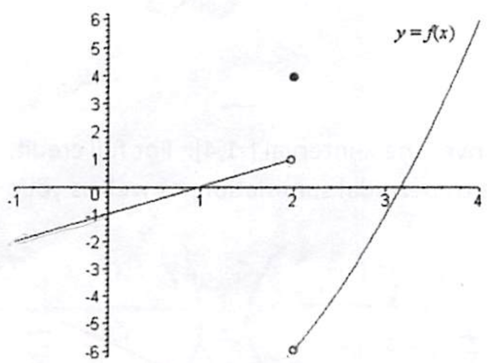
$x_1 = 1.9949849$ $\quad$ $x_2 = 2.0049851$

$|2 - x_1| > |2 - x_2|$, use x_2

x-value should be .0049851 w/in 2 in order to be w/in .05 of L 7

Ok, stop referring to $f(x) = x^3 - 2x + 3$. We're moving on to other stuff.

9. Given the graph of f(x) below,



a) $\lim_{x \to 2^+} f(x) =$ 1

b) $\lim_{x \to 2^-} f(x) =$ −6

c) $f(2) =$ 4

d) $\lim_{x \to 2} f(x) =$ DNE

10. On the axes below, sketch a single graph of g(x) that has ALL of the following characteristics:

$\lim_{x \to 3} f(x) = 4$ ✓

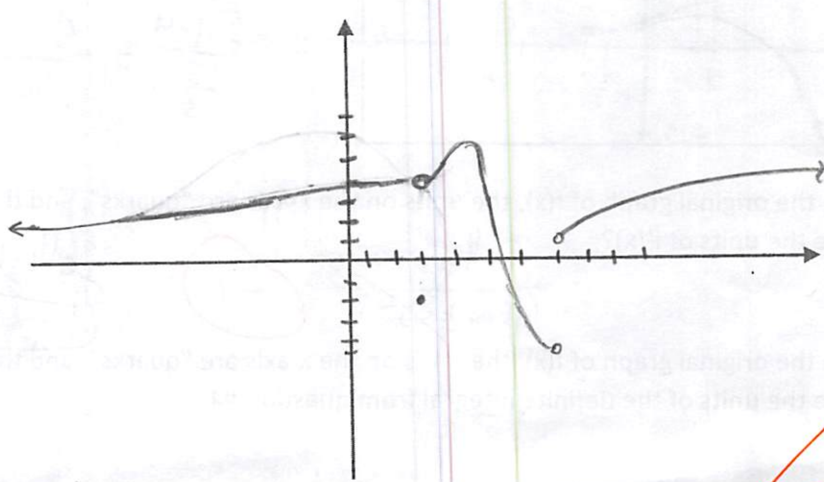
$f(3) = -2$ ✓

$\lim_{x \to 7^-} f(x) = -5$ ✓

$\lim_{x \to 7^+} f(x) = 1$ ✓

$f'(4) > 0$ ✓

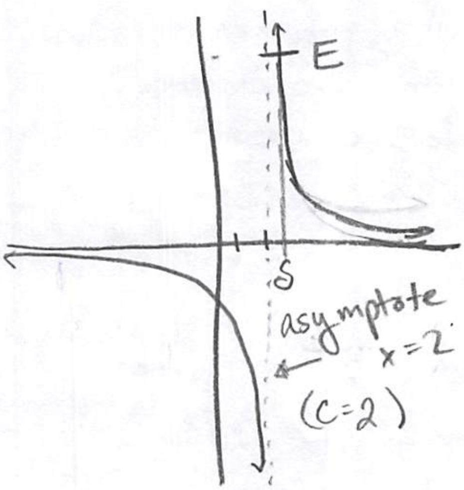
$\lim_{x \to -\infty} f(x) = 2$ ✓



1-0

11. Use some combination of L, E, D, δ, ε, c, and x (but not all of them, obvi) to prove the limit. In your work, include a labeled graph, algebraic work, and a conclusion statement.

Prove: $\lim\limits_{x \to 2} \frac{x+4}{x-2} = \infty$



dist. error

$\frac{x+4}{x-2} = E$

$x+4 = Ex - 2$ -1

$x - Ex = -6$

$x(1-E) = -6$

$x_1 = \frac{-6}{1-E}$

$\delta = \frac{-6}{1-E} - 2$

$= \frac{-6}{1-E} + \frac{-2(1-E)}{1-E}$

$= \frac{-6-2+2E}{1-E} = \frac{-8+2E}{1-E}$

when $|x-c| < -\frac{8+2E}{1-E}$ for a large number E, $\frac{x+4}{x-2} > E$ and does not leave this neighborhood, thus $\lim\limits_{x \to 2} \frac{x+4}{x-2} = \infty$

12. f(x) is continuous at x = c if:

(i) f(c) exists

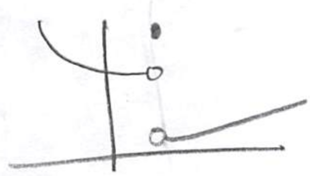
(ii) $\lim\limits_{x \to c} f(x)$ exists

(iii) $\lim\limits_{x \to c} f(x) = f(c)$

For each part below, sketch a function that is NOT continuous, because it only upholds the given statements from the definition of continuity.

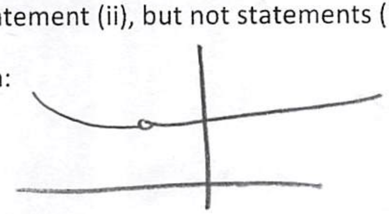
a) upholds statement (i), but not statements (ii) and (iii)

Sketch:



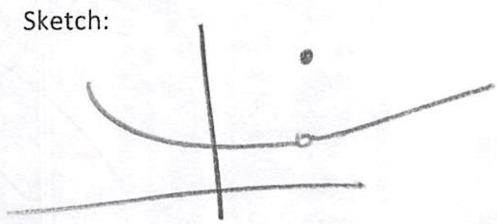
b) upholds statement (ii), but not statements (i) and (iii)

Sketch:



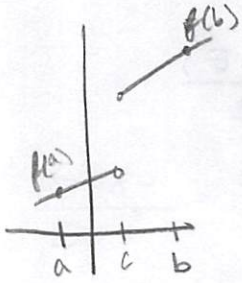
c) upholds statements (i) and (ii) but not statement (iii)

Sketch:



-1

13. Explain why the Intermediate Value Theorem doesn't always apply to a function that is NOT continuous. Include a sketch in your explanation.



]No f(c), and large jump between c-δ & c+δ

IVT doesn't always apply to non-cont. functions bc not all y-values between f(a) & f(b) need to be covered. For example, take point "c". c doesn't exist, but c-δ (δ = small #) & c+δ do. f(c-δ) & f(c+δ) are separated by a large jump in value. This leaves a gap in y-values that aren't hit anywhere between a & b, which means the IVT may or may not work depending on the chosen point.

14. Given $f(x) = \frac{1}{x}$ use the Formal Definition of Derivative to find $f'(x)$.

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)+f(x)}{h} = \lim_{h\to 0} \frac{\frac{1}{x+h} + \frac{1}{x}}{h} = \lim_{h\to 0} \frac{\frac{hx + x+h}{x(x+h)}}{h}$$

$$= \lim_{h\to 0} \frac{\frac{h(2x+h)(hx-x^2)}{(x^2+hx)(hx-x^2)}}{h} = \lim_{h\to 0} \frac{\frac{2hx^2+h^2x-2x^3-hx^2}{h^2x^2-x^4}}{h} = \lim_{h\to 0} \left(\frac{-2x^2+hx+h^2}{h^2x-x^3}\right)\left(\frac{1}{h}\right)$$

$$= \lim_{h\to 0} \frac{-\frac{2x^2}{h} + x + h}{h^2x - x^3} = \frac{x}{-x^3} = \boxed{-\frac{1}{x^2}}$$

$$f(x) = \frac{1}{x} = x^{-1} \qquad f'(x) = -x^{-2} = -\frac{1}{x^2}$$