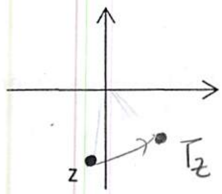
Analysis H – Deggeller/Hahn
GAtM Quiz 3 2016-2017
NO CALCULATORS
[39 points, + 1 for writing your name]

38/40

Composed Student: Hannah Kim
Period: ______

1. Consider the transformation matrix $T = \begin{bmatrix} \cos\frac{2\pi}{7} & -\sin\frac{2\pi}{7} \\ \sin\frac{2\pi}{7} & \cos\frac{2\pi}{7} \end{bmatrix}$, and point z (shown on the axes below)

   a) Draw the image of Tz on the same axes, and describe the effect of matrix T (be specific) [3 pts]



Effect of Matrix T: rotation by $\frac{2\pi}{7}$

   b) What is the period of T? [1] 7

   c) T is the generator of the group G. Name a group that is isomorphic to group G. [2]

   rotation group of a 7-sided regular polygon

2. Consider the transformation matrix $R = \begin{bmatrix} \cos\frac{\pi}{2} & \sin\frac{\pi}{2} \\ \sin\frac{\pi}{2} & -\cos\frac{\pi}{2} \end{bmatrix}$, and point w (shown on the axis below).

   a) Draw the image of Rw on the same axes, and describe the effect of matrix T (be specific) [3]

Effect of Matrix R: flip over line $\theta = \frac{\pi}{4}$

   b) What is the period of R? [1] 2

   c) R is the generator of the group H. Name a group that is isomorphic to group H. [2]

   flip group of line

3. Matrix T (from problem 1) and Matrix R (from problem 2), taken together, generate group J. [3]

   Group J has 14 elements, and is isomorphic to reflection group of 7-gon.

4. Matrix R is from problem 2, and Matrix S is a reflection across the x-axis. What is the effect of Matrix SR? As always, be specific. [3]

rotation by $-\frac{\pi}{4}$ -1

$$\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} R \end{bmatrix} = \begin{bmatrix} -\cos\frac{\pi}{2} & -\sin\frac{\pi}{2} \\ \sin\frac{\pi}{2} & -\cos\frac{\pi}{2} \end{bmatrix}$$

-1

$$\begin{bmatrix}0&0&-1\\0&1&0\\1&0&0\end{bmatrix}\begin{bmatrix}-1&0&0\\0&1&0\\0&0&-1\end{bmatrix}=\begin{bmatrix}0&0&1\\0&1&0\\-1&0&0\end{bmatrix}\Rightarrow\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}\quad\begin{bmatrix}0&0&-1\\0&1&0\\1&0&0\end{bmatrix}\begin{bmatrix}0&0&-1\\0&1&0\\1&0&0\end{bmatrix}=\begin{bmatrix}-1&0&0\\0&1&0\\0&0&-1\end{bmatrix}$$

5. Consider the following transformation matrices in **3-D**:

$$A=\begin{bmatrix}0&0&-1\\0&1&0\\1&0&0\end{bmatrix}\qquad B=\begin{bmatrix}0&0&1\\0&1&0\\1&0&0\end{bmatrix}\qquad C=\begin{bmatrix}3&0&0\\0&1&0\\0&0&1\end{bmatrix}$$

$$\begin{bmatrix}0&0&-1\\0&1&0\\1&0&0\end{bmatrix}\begin{bmatrix}0&0&1\\0&1&0\\ &0&0\end{bmatrix}\qquad \begin{matrix}-1&0&0\\0& &0\\0&0&1\end{matrix}$$

a) How many elements are in the group generated by… [2 each]

A alone? 4

B alone? 2

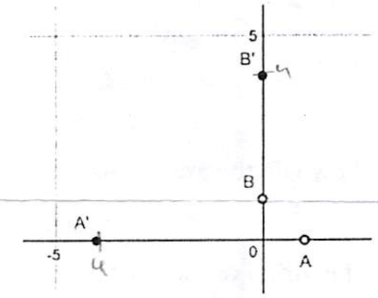
A and B together? 8

b) Describe the effect of matrix C. Would C generate a group? Justify your answer. [3]

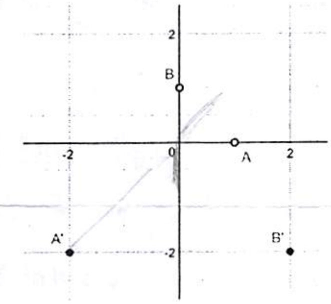
stretch x by 3. It wouldn't generate a group because it wouldn't create any way to reverse it. $\begin{bmatrix}3&0&0\\0&1&0\\0&0&1\end{bmatrix}$ & $\begin{bmatrix}\frac{1}{3}&0&0\\0&1&0\\0&0&1\end{bmatrix}$ would generate a group tho.

6. Below are examples of a unit square preimage (A and B) being transformed into an image (A' and B'), after being transformed 2 times in succession. For each picture, write the 2 matrices (in the correct order) that, when multiplied together, accomplish the depicted transformation. [3 pts each]

a)



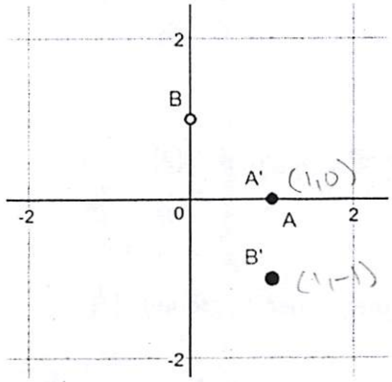
b)



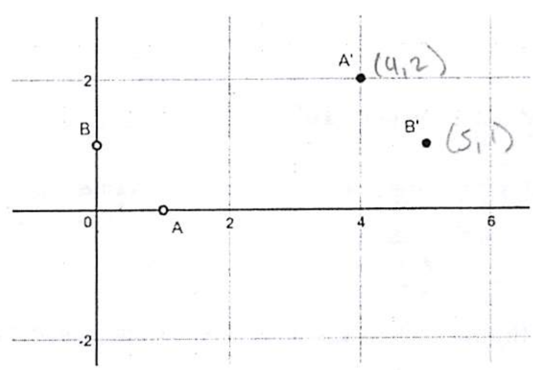
a) Answer: $\begin{bmatrix}4&0\\0&4\end{bmatrix}\cdot\begin{bmatrix}\cos\pi&\sin\pi\\\sin\pi&-\cos\pi\end{bmatrix}$

b) Answer: $\begin{bmatrix}2\sqrt{2}&0\\0&2\sqrt{2}\end{bmatrix}\cdot\begin{bmatrix}\cos\frac{5\pi}{4}&-\sin\frac{5\pi}{4}\\\sin\frac{5\pi}{4}&\cos\frac{5\pi}{4}\end{bmatrix}=\begin{bmatrix}-2&2\\-2&-2\end{bmatrix}$

c)



d)



c) Answer: $\begin{bmatrix}1&0\\0&-1\end{bmatrix}\cdot\begin{bmatrix}1&1\\0&1\end{bmatrix}$

d) Answer: $\cancel{\begin{bmatrix}4&0\\2&1\end{bmatrix}}\cdot\cancel{\begin{bmatrix}1&\frac{5}{4}\\0&-\frac{3}{2}\end{bmatrix}}$ -1

$\begin{bmatrix}1&1\\0&-1\end{bmatrix}$

$$\begin{bmatrix}4&0\\0&1\end{bmatrix}\begin{bmatrix}1&0\\2&1\end{bmatrix}\begin{bmatrix}1&0\\0&-\frac{3}{2}\end{bmatrix}\begin{bmatrix}1&\frac{5}{4}\\0&1\end{bmatrix}\begin{bmatrix}1&-\frac{5}{4}\\0&1\end{bmatrix}\begin{bmatrix}-1&0\\0&\frac{2}{3}\end{bmatrix}\begin{bmatrix}1&0\\-2&1\end{bmatrix}\begin{bmatrix}\frac{1}{4}&0\\0&1\end{bmatrix}=\begin{bmatrix}4&5\\2&1\end{bmatrix}$$

-1