

28/30

1. In Trig last year you learned that the period of the function $y = 3\cos(12x)$ is $\frac{\pi}{6}$ and that the amplitude is 3. Explain how both of these connect to the graph of $r = 3\cos(12\theta)$ in polar. [3]

The amplitude, 3, represents the height of the rectangular wave & length of the polar petal. Period, in both of them, is the distance & rotation needed to make a full wave of petal. It then repeats itself.

amp 3 ✓ period $\frac{\pi}{6}$ ✓ -1

$\frac{2\pi}{12} = \frac{\pi}{6}$

2. Consider the polar rose $r = 5\sin(15\theta)$

✓ a) How many visible petals will there be? 15 [2]

✓ b) Will there be a petal along the positive y axis, negative y-axis, both or neither? negative y [2]

$$r = 5\sin(15 \cdot \frac{\pi}{2})$$

$$= 5\sin(\frac{15\pi}{2}) = 5\sin(\frac{3\pi}{2}) = -5$$

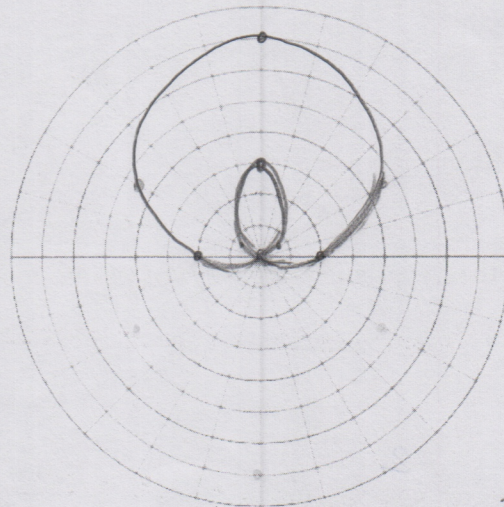
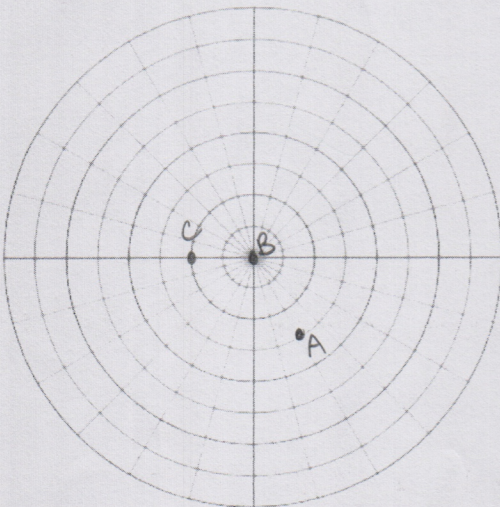
$(-5, \frac{\pi}{2})$ and $(5, \frac{3\pi}{2})$ are the same point

$$r = 5\sin(15 \cdot \frac{3\pi}{2}) = 5\sin(\frac{45\pi}{2}) = 5\sin(\frac{\pi}{2}) = 5$$

✓ c) The line of symmetry for the first petal in Quadrant I, is at $\theta = \frac{\pi}{30}$ [2]

✓ 3. On the polar graph paper below left plot and label the 3 points: [3]

A: $(-3, -\frac{4\pi}{3})$ B: $(0, \frac{\pi}{3})$ C: $(-2, 0)$



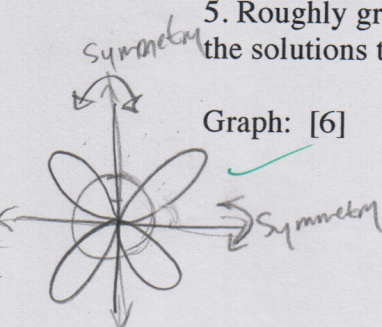
✓ 4. Above right, accurately graph $r = -2 + 5\sin\theta$. [5]

$$r = -2 + 5\sin(\frac{3\pi}{2})$$

$$-7 = -2 + -5$$

$$\begin{aligned} -2 + 5\sin(-\frac{\pi}{2}) &= -2 + 5 \cdot -1 = -7 \\ -2 + 5 \cdot -\frac{1}{2} &= -2 - \frac{5}{2} = -\frac{9}{2} \\ -2 + -\frac{5}{2} &= -\frac{9}{2} \end{aligned}$$

5. Roughly graph the system of equations $r = 2$ and $r = 4\sin 2\theta$ below, and clearly state all of the solutions to the system as ordered pairs in numerical order (in radians). Show all work.



Graph: [6]

$$2 = 4\sin 2\theta$$

$$\frac{1}{2} = \sin 2\theta$$

$$2\theta = \frac{\pi}{6}$$

$$2\theta = \frac{5\pi}{6}$$

$$\theta = \frac{\pi}{12}$$

$$\theta = \frac{5\pi}{12}$$

$$-\frac{\pi}{12}$$

$$-\frac{5\pi}{12}$$

$$\pi - \frac{\pi}{12} = \frac{11\pi}{12}$$

$$\frac{7\pi}{12}$$

$$\frac{11\pi}{12}$$

$$-\frac{7\pi}{12}$$

$$-\frac{11\pi}{12}$$

flip this a lot

Solutions: $(2, \frac{\pi}{12}), (2, \frac{5\pi}{12}), (2, \frac{7\pi}{12}), (2, \frac{11\pi}{12}), (2, \frac{13\pi}{12}), (2, \frac{17\pi}{12}), (2, \frac{19\pi}{12}), (2, \frac{23\pi}{12})$

6. Convert the point $(-5, -5)$ into polar coordinates in radians. [2]

$$(\sqrt{(-5)^2 + (-5)^2}, \tan^{-1}(\frac{-5}{-5}))$$

$$(5\sqrt{2}, \frac{5\pi}{4})$$

7. Convert the point $(r, \theta) = (-2, 5\pi/6)$ into simplified rectangular form. [2]

$$(-2 \cos \frac{5\pi}{6}, -2 \sin \frac{5\pi}{6})$$

$$(-2 \cdot \frac{\sqrt{3}}{2}, -2 \cdot \frac{1}{2})$$

$$(\sqrt{3}, -1)$$

8. Convert the hyperbola $r^2 = 20/\sin 2\theta$ into rectangular. [3]

$$r^2 = \frac{20}{\sin 2\theta}$$

$$r^2 = \frac{20}{2\cos\theta\sin\theta}$$

$$r^2 \cos\theta \sin\theta = 20$$

$$xy = 20$$

$$y = \frac{20}{x}$$

-1

1-1