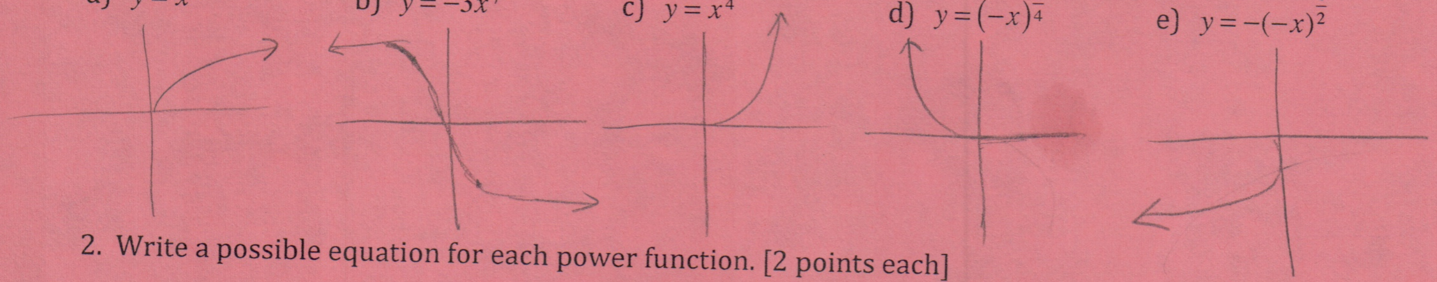
Analysis H – Deggeller / Hahn
Unit 5 – Growth Quest
NO CALCULATOR SECTION

$\frac{48}{50}$

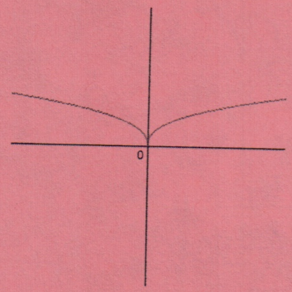
Power Player: Hannah Kim
Period: B

✓ 1. Graph each power function (thumbnail sketch is ok - just grading on curvature and quadrants). [2 ea]

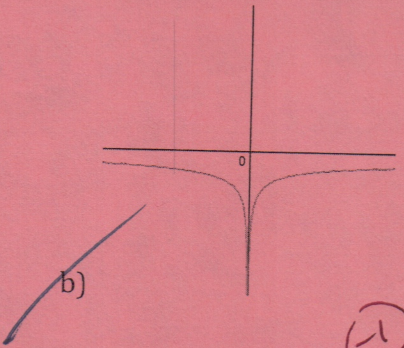
a) $y = x^{\frac{5}{6}}$ b) $y = -3x^{\frac{1}{7}}$ c) $y = x^{\frac{9}{4}}$ d) $y = (-x)^{\frac{9}{4}}$ e) $y = -(-x)^{\frac{1}{2}}$



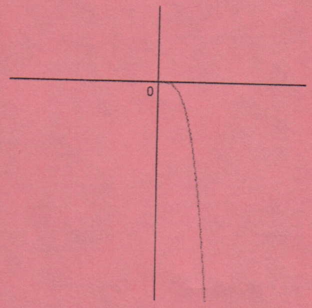
2. Write a possible equation for each power function. [2 points each]



a)



b)



c)

$y = x^{\frac{2}{3}}$

−1

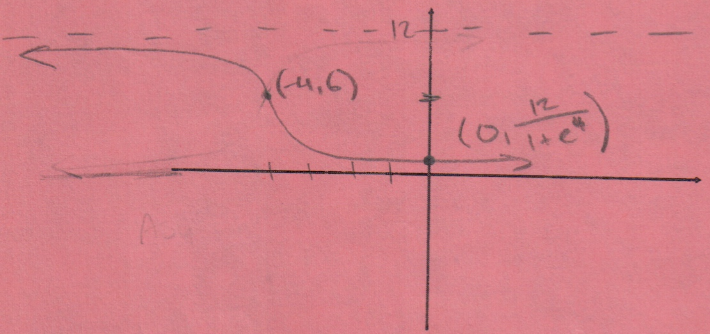
$y = x^{\frac{2}{3}} - 5$

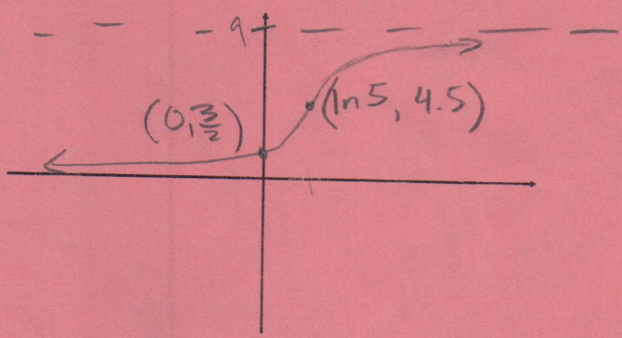
$y = -x^{\frac{9}{4}}$

3. Graph the function. For each, clearly graph and label the asymptotes, y-intercept, and point of inflection. [3 each]

✓ a) $y = \dfrac{12}{1+e^{x+4}}$

b) $y = \dfrac{18}{2+10e^{-x}} = \dfrac{9}{1+5e^{-x}}$





asymptotes: $y = 12,\ y = 0$

y-intercept: $\left(0, \dfrac{12}{1+e^4}\right)$

point of inflection: $(-4, 6)$

asymptotes: $y = 9,\ y = 0$

y-intercept: $\left(0, \dfrac{9}{6}\right) = \left(0, \dfrac{3}{2}\right)$

point of inflection: $(\ln 5, 4.5)$

ok

−1

4. Solve for x. [4 each]

a) $\log_{16}(x+1)^2 + \log_{16}(x+1) = \log_{16} 64$

$\cancel{\log_{16}}(x+1)^3 = \cancel{\log_{16}} 64$

$(x+1)^3 = 64$

$(x+1)^3 = 4^3$

$(x+1) = 4$

$\boxed{x = 3}$ ✓

$2 + \ln\sqrt{1+x} + \cancel{3}\ln\sqrt{(1-x)^3} = \ln\sqrt{1-x^2}$

$2 = \ln\left(\frac{\sqrt{1-x^2}}{\sqrt{1+x}\ \sqrt{(1-x)^3}}\right)$

$2 = \frac{1}{2}\ln\left(\frac{1-x}{(1+x)(1-x)^3}\right)$

$4 = \ln\left(\frac{1}{(1-x)^2}\right)$

$\frac{1}{(1-x)^{\cancel{2}}} = e^{\cancel{4}2}$

$\frac{1}{e^2} = 1-x$

$x = 1 - \frac{1}{e^2}$

b) $2 + \ln\sqrt{1+x} + 3\ln\sqrt{1-x} = \ln\sqrt{1-x^2}$

$2 + \frac{1}{2}\ln(1+x) + \frac{3}{2}\ln(1-x) = \frac{1}{2}\ln(1-x^2)$

$4 + \ln(1+x) + 3\ln(1-x) = \ln(1-x^2)$

$4 = \ln(1-x^2) - \ln(1+x) - 3\ln(1-x)$

$4 = \ln\left(\frac{\cancel{1-x^2}\ 1}{\cancel{(1+x)}(1-x)^{\cancel{3}2}}\right)$

$4 = \ln\frac{1}{(1-x)^2}$

$e^{\cancel{4}2} = \frac{1}{(1-x)^{\cancel{2}}}$

$e^2 - e^2 x = 1$

$\boxed{x = 1 - \frac{1}{e^2}}$ ✓

$x = -e^{-2}$

CALCULATOR SECTION Smart Investor: Hannah Kim

5. A certain population, $P(t)$, of wombats studied from 1995 to 2015 was found to be growing according to a logistic model where t = years after 2000. The point of inflection of the graph is at $t = -4$, and the carrying capacity for the region is 800.

a) Given that the domain of this function is all real numbers, express the number of wombats as a function of t (where t is years after 2000). There are many different answers that work here so make your life (and your teacher's life) easier by picking the simplest one. [4 pts]

$$y = \frac{800}{1+ae^{-bt+c}}$$

$$\boxed{y = \frac{800}{1+e^{-t-4}}}$$ ✓

$$1 = \frac{2}{1+e^{4-4}}$$

$$1 = \frac{2}{2}$$

b) What was the population in 1998 (round to the nearest wombat)? [2 pts]

$$y = \frac{800}{1+e^{-(-2)-4}}$$

$$y = 704.637$$

$$y = \frac{800}{1+\frac{1}{e^2}}$$

$$\boxed{y = 705 \text{ wombats}}$$ ✓

c) When were there 300 wombats? Give your answer as a month and a year. [2 pts]

$$\cancel{300} = \frac{\cancel{800}}{1+e^{-t-4}}$$

$$\boxed{\text{June } 199\cancel{6}}$$

-1

$$8 = 3+3e^{-t-4}$$

$$\frac{5}{3} = e^{-t-4}$$

$$\ln\frac{5}{3} = -t-4$$

$$t = -\ln\left(\frac{5}{3}\right) - 4$$

$$t = -4.51$$

$$2000 + -4.51 = 1996,$$

June | July

.5

-1

$$FV = C \cdot \left[\frac{(1+i)^n - 1}{i} \right] \qquad PV = C \cdot \left[\frac{1-(1+i)^{-n}}{i} \right]$$

6. *How much house can you afford?* The rate on a 30-year home loan is currently around 4.25%. If you can afford \$3000 monthly payments for the next 30 years,

a) How much money can you borrow towards the purchase of a house? [4 pts]

$$PV = 3000 \left[\frac{1 - \left(1 + \frac{.0425}{12}\right)^{-360}}{\frac{.0425}{12}} \right] = \boxed{\$609,830.60} \checkmark$$

b) Assuming you borrow the maximum amount from part (a): Over the course of the loan repayment, how much of your total payment was spent on interest? [2 pts]

$$3000 \cdot 360 = 1080000 - 609,830.60$$

$$= \boxed{\$470169.40} \checkmark$$

7. *Who wants to be a millionaire!?* Brain decides to get serious about saving for his future. Starting this month, he promises to deposit \$850 into a stock market account, which will give him a 7% annual return. How long will it be before Brian has grown his savings to \$1,000,000? To receive credit, clearly show your solution strategy and work involved (including how you used your calculator). [4 pts]

$n = \#$ of months

$$100000\cancel{0} = 85\cancel{0} \left[\frac{\left(1 + \frac{.07}{12}\right)^{n} - 1}{\frac{.07}{12}} \right]$$

$$\frac{100000}{85} \cdot \frac{12}{.07} = \left(1 + \frac{.07}{12}\right)^{n} - 1$$

$$201680.6723 = (1.00583)^{n} - 1$$

$$201681.6723 = \left(1 + \frac{.07}{12}\right)^{n}$$

$$n = \log_{\left(1 + \frac{.07}{12}\right)} (201681.6723)$$

$$\frac{\log(201681.6723)}{\log\left(1 + \frac{.07}{12}\right)} = n$$

$$n = 2100.01 \qquad \frac{2100}{12} = 175$$

~~175 years~~

$\boxed{27.58 \text{ years or } 27 \text{ years } \& \text{ } 7 \text{ months}}$

← disregard this; I don't have an eraser.

-0

$$100000\cancel{0} = 85\cancel{0}\left[\frac{\left(1+\frac{.07}{12}\right)^n - 1}{\frac{.07}{12}}\right]$$

$$\frac{100000 \cdot 12}{85\cancel{0} \cdot .07} = \left(1+\frac{.07}{12}\right)^n - 1$$

$$\frac{(100000)(12)}{(85\cancel{0})(.07)} + 1 = \left(1+\frac{.07}{12}\right)^n$$

$$n = \frac{\log\left(\frac{(100000)(12)}{(85)(.07)}\right)}{\log\left(1+\frac{.07}{12}\right)}$$

$$n = 2100$$

This is it (I hope lol)

$$100000\cancel{0} = \frac{85\cancel{0}\left(1-\left(1+\frac{.07}{12}\right)^n\right)}{\left(1-\left(1+\frac{.07}{12}\right)\right)}$$

$$100000 = \frac{85\left(1-\left(1+\frac{.07}{12}\right)^n\right)}{\left(1-\left(1+\frac{.07}{12}\right)\right)}$$

$$1 - \frac{(100000)\left(1-\left(1+\frac{.07}{12}\right)\right)}{85} = \left(1+\frac{.07}{12}\right)^n$$

$$n = \frac{\log\left(\frac{(100000)\left(1-1+\frac{.07}{12}\right)}{85}\right)}{\log\left(1+\frac{.07}{12}\right)}$$

$$n = 331.15$$

$$331 \div 12 = 27.58$$