

No Calculators on this test. But no reason to simplify your answers either.

Questions 1-4 are Multiple Choice. Circle the best answer. [3 each]

1. Which of the following expressions are equivalent to entry $\binom{17}{8}$ in Pascal's Triangle?

I. $\binom{16}{7} + \binom{16}{8}$

II. $\binom{18}{9} - \binom{17}{9}$

III. $\binom{8}{0} + \binom{9}{1} + \binom{10}{2} + \dots + \binom{16}{8}$

a) I only

b) I and II only

c) I and III only

d) II and III only

e) I, II, and III.

2. $\frac{(n+2)! - n!}{(n+1)!}$ can be factored into a rational function in the form $\frac{ax^2 + bx + c}{dx + e}$. Find the sum $a + b + c + d + e$.

a) 8

b) 3

c) 7

d) 9

e) 5

3. As n gets bigger and bigger (goes towards infinity), then the following sum will approach what value?

$$\sum_{k=1}^n 3\left(\frac{2}{5}\right)^k$$

a) 7

b) 7.5

c) 2

d) 5

e) 1.2

4. Use telescoping to derive a compact expression for the following sum of even-numbered Fibonacci terms:

$$F_{14} + F_{16} + \dots + F_{200}$$

a) $F_{201} - F_{13}$

b) $F_{202} - F_{12}$

c) F_{203}

d) F_{202}

e) $F_{202} - F_{13}$

5. The number 28 can be found in 7 locations in Pascal's Triangle (and the negative Pascal's Triangle). 2 such locations are $\binom{28}{1}$ and $\binom{28}{27}$ but those are boring. Express 28 as 4 **different** binomial coefficients. [4]

$$28 = (\quad) = (\quad) = (\quad) = (\quad)$$

6. Write the following sum using sigma notation. Then actually calculate the sum (in terms of "m") [5]

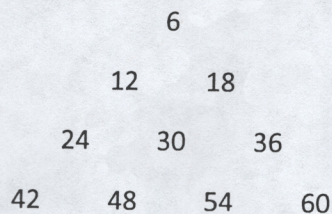
$$5 + 11 + 17 + 23 + \dots (6m - 19)$$

Sigma: _____

Sum: _____

7. Find the coefficient for the $x^{10}y^{25}z^{15}$ term in the expansion of $(3x + 2y + z)^{50}$ [4]

8. Consider the "triangle of 6's" below. The last row shown is the 4th row. The first term of the n th row can be found by the formula. $F(n) = 3n^2 - 3n + 6$



a) Find an expression for the middle term in the n th row (where n is an odd number). [3]

b) Write a compact expression for the product of the first n terms in the triangle above $(6)(12)(18).....$ using factorials and/or exponents. Your answer will have n in it. [2]

9. The method of induction can be used to prove the following statement:

"The expression $a^2 - 1$ is divisible by 8 for all positive odd numbers a "

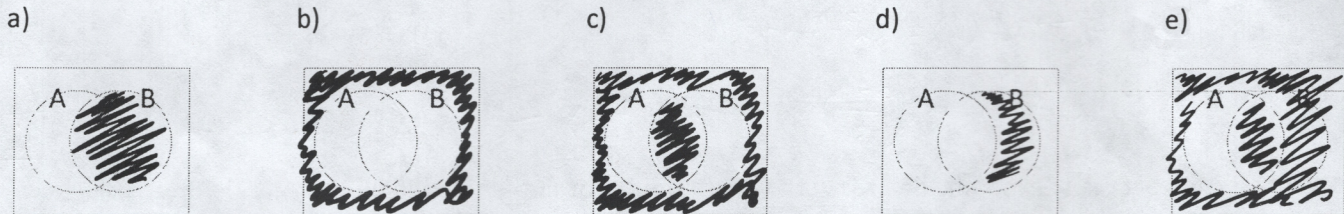
Properly right out the first three steps in a potential induction proof. **YOU DO NOT NEED TO DO THE ENTIRE PROOF!!!**
Please properly label all 3 steps. [5]

Questions 9-13 are Multiple Choice (Again!?) [3 each]

10. How many ways can you split 7 students into 2 groups, where each group has at least one student?

- a) 7! b) 128 c) 126 d) 63 e) 64

11. Which diagram represents $P(A' \cup B)$?



12. Which is logically equivalent to $P(A \cup B)'$?

- a) $P(A' \cap B)$ b) $P(A \cap B')$ c) $P(A' \cap B)'$ d) $P(A' \cap B')$ e) $P(A \cap B)$

13. How many distinct 3-letter arrangements can you make from the letters in the word "COLTS"?

- a) 33 b) 24 c) 60 d) 120 e) 30

14. How many distinct 3-letter arrangements can you make from the letters in the word "CALLS"?

- a) 33 b) 24 c) 60 d) 120 e) 30

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15. 7 students randomly arrange themselves into a circle. What is the probability that Ed is standing directly between Edd and Eddy? (obviously assuming that Ed, Edd, and Eddy are 3 of the 7 students) [3]

16. Jar A contains 2 white and 2 blue marbles. Jar B contains 1 white and 2 blue marbles.

a) A random jar is selected, and then a random marble is taken out of the jar. What is the probability that the marble is blue? [3]

b) A random jar is selected, and then a random marble is taken out of the jar. What is the probability that Jar A was selected, given that the marble is blue? [3]

c) A random marble is selected out of Jar A and placed into Jar B. Then a random marble is selected from Jar B. What is the probability that a blue marble was taken out of Jar A, given that the final marble is blue? [3]

17. I have 2 nickels and 3 quarters in my pocket.

a) If I randomly choose 2 of the coins, what is the probability that I will select one nickel and one quarter? [3]

b) If I randomly choose 2 of the coins, what is the expected value of the two coins together? [4]

18. In order to gain access to the exclusive We Love Ones Club, you must show your love for 1's by rolling 6 fair, 6-sided dice, and getting at least 2 of dice to show a "1". What is the probability that you will gain access? [4]

19. What is the probability of being dealt a 7 card hand in poker (assume a 52 card deck) and getting a Full House (3 of one denomination, 2 of another and 2 "other" cards)? [4]