Analysis H 2018-19 – Deggeller / Gleason / Hahn
Midterm 2 **No Calculators!!** [101 points]

The Midterminator II: _____
Per: _____

PART 1: Polar and 3D graphing

For problems 1-8, match each of the 3-d curves with their name. Each letter may be used more than once, or not at all. [2 pts each]

A) plane
B) hyperboloid of one sheet
C) hyperboloid of two sheets
D) elliptic paraboloid
E) elliptic cone
F) ellipsoid
G) hyperbolic paraboloid (saddle)
H) A different curve, not listed in A-G

1. $-5x + y^2 - 2z^2 = 10$ _____
2. $-x^2 + y^2 - 4z^2 = 12$ _____
3. $3x^2 + 3y^2 - 5z^2 = 0$ _____
4. $x + 2(y + 3)^2 + 4z^2 = 28$ _____
5. $-5x + y - 2z = 10$ _____
6. $-x^2 - y^2 - 4z^2 = 12$ _____
7. $3x^2 + 3y^2 - 5z^2 = 11$ _____
8. $x = z$ _____

9. Consider the graph of $r = 3 - 2\sin\theta$. Circle **ALL** of the statements below that are true. [6]

I. It is a limacon
II. It has a dimple
III. It's symmetric about the x axis
IV. It's symmetric about the y axis
V. It's max r – value is 5
VI. It has an inner loop

10. Which of the following is an equation of a rose curve with 10 petals? (circle 1 answer) [3 pts]

a) $r = 5\cos(10\theta)$
b) $r = 5\cos(5\theta)$
c) $r = 5\sin(5\theta)$
d) $r = 5\sin(10\theta)$
e) None of these

11. The traces of a hyperboloid of 2 sheets are: (circle 1 answer) [3]

a) two hyperbolas and one parabola
b) one hyperbola and two parabolas
c) two hyperbolas and one ellipse
d) one hyperbola and two ellipses
e) none of these

Polar and 3D Free Response:

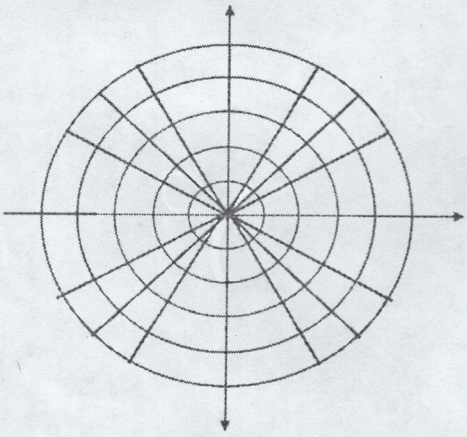
12. Sketch the cylindrical point $(r, \theta, z) = \left(1, -\frac{\pi}{6}, -1\right)$ and then convert it into spherical coordinates. [4]

Rough Sketch:

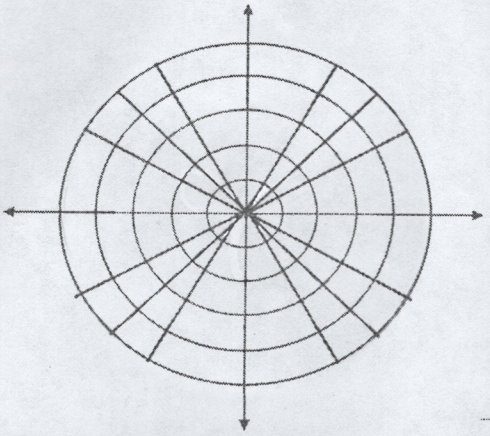
$(\rho, \theta, \phi) =$ _____

13. Quickly but accurately graph each polar curve below. [3 pts each]

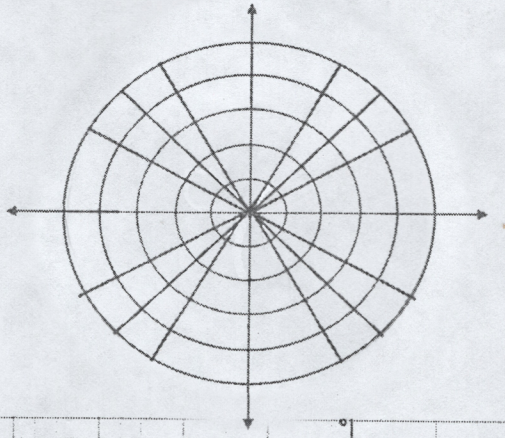
a) $r = 2\sin 3\theta$



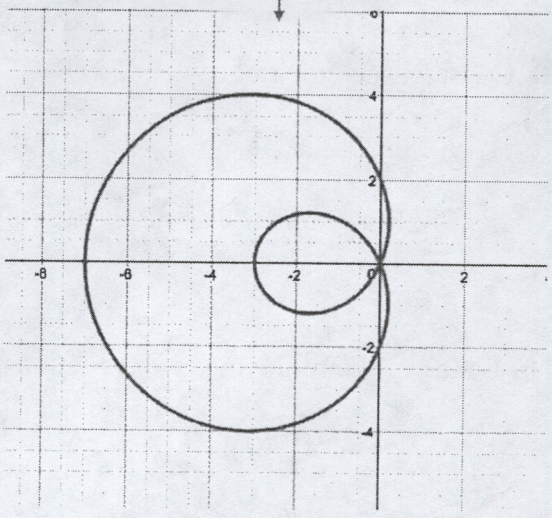
b) $r = 4 - \cos\theta$



c) $r^2 = 9\sin 2\theta$



14. Write the equation of the graph on the right in polar form. [4]

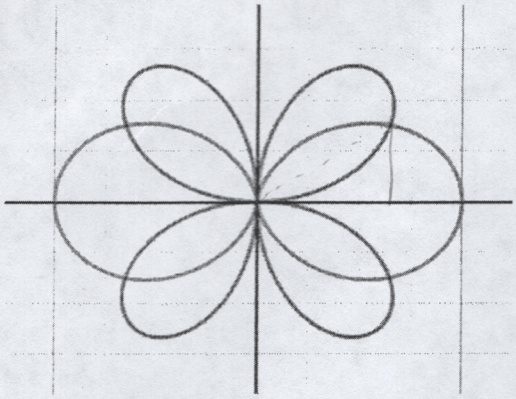


_____ = _____

15. Convert $x^2 + y^2 = 2\sqrt{x^2 + y^2} - 2y$ to **polar** form, and then identify the shape by its most specific name. [4]

Polar form: _____ Name: _____

16. The curves $r = 2\cos^2\theta$ and $r = \sqrt{3}\sin 2\theta$ (graphed below) cross 5 times. Find two of the intersection points and write them in polar form. Show the algebra that leads to your answer for full credit. [5]



point 1 _____ point 2 _____

17. Any ellipsoid can be written in the form $\frac{(x-a)^2}{d}+\frac{(y-b)^2}{e}+\frac{(z-c)^2}{f}=1$.

Create an ellipsoid that has its center at (1, 0, 0) and x-intercepts 4 and –2. Also make it have y-intercepts ±5, and pass through the point (3,0,1). [4]

PART 2: Vectors and Parametric Equations

18. **Multiple Choice:** The graph of the set of parametric equations $\begin{aligned} x(t) &= \cos t \\ y(t) &= 3-\sin^2 t \end{aligned}$ is a _____. [3]

a) circle b) ellipse c) parabola d) hyperbola e) spiral

19. Vector **a** is drawn below. Draw and label another vector **b** such that… [4]

a) $\boldsymbol{a} \times \boldsymbol{b}$ would have a direction **up** perpendicular out of this piece of paper.
b) The scalar projection $proj_b a < 0$

a

20. Find the equation of the plane, in standard Ax+By+Cz+D = 0 form, that contains the following 3 non-collinear points: [6]

(2, 0, 3) (3, -1, 1) and (0, 4, 4)

21. Grayson is launching a grapefruit off the top of a building. The position (in feet) of the grapefruit after t seconds is given by the set of parametric equations:

$$x(t) = 40t\sqrt{3}$$
$$y(t) = 40t - 16t^2 + 70$$

Answer the following series of short answer questions about this scenario. [8]

a) True or False: The grapefruit is launched from an initial height of 70 feet off the ground. _____

b) The grapefruit was launched at a velocity of _____ f/s at an angle of _____ degrees

c) After 1 second, the grapefruit is at a height of _____ feet.

d) The 2nd time the grapefruit will be 70 feet off the ground is at _____ seconds.

22. Consider the two vectors $\mathbf{r} = \langle 6, 8, 0 \rangle$ and $\mathbf{s} = \langle 2, 2, -1 \rangle$ Fill in the blanks below either with <, >, = or NEI (not enough information) [2/2/2/2/4]

a) $\boldsymbol{r \cdot s}$ _____ 28

b) The angle between the two vectors _____ 60 degrees

c) $scalar\ proj_s r$ _____ $|\boldsymbol{s}|$

d) The area of the parallelogram formed by the two vectors _____ 10 square units

e) Now, using the same vectors **r** and **s**, calculate the distance from the point (-3, -2, 1) to the plane formed by vectors **r**, **s** and the origin.

23. Name a plane (in standard form) perpendicular to the plane $3x - 5y + 2z = 20$. Then using words and math, convince me that your answer is correct. Many answers are possible. [4]

Your plane _____

Your argument:

24. Consider line L: $\langle x,y,z \rangle = \langle -2,5,1 \rangle + \langle 1,2,-4 \rangle t$

a) Is the point (98, 205, -350) on line L? Justify your answer. [3]

Yes or no: _____

Justification:

b) Line L above intersects this new line $\langle x,y,z \rangle = \langle 2,-2,27 \rangle + \langle 3,1,2 \rangle t$
Find the point of intersection of the two lines. [3]

(x,y,z) = _____