sis H 2018-19 – Deggeller / Gleason / Hahn — The Midterminator II: Timothy Herhen

term 2 **No Calculators!!** [101 points] — Per: 5

97/101

## PART 1: Polar and 3D graphing

For problems 1-8, match each of the 3-d curves with their name. Each letter may be used more than once, or not at all. [2 pts each]

A) plane  B) hyperboloid of one sheet  C) hyperboloid of two sheets

D) elliptic paraboloid  E) elliptic cone  F) ellipsoid

G) hyperbolic paraboloid (saddle)  H) A different curve, not listed in A-G

1. $-5x + y^2 - 2z^2 = 10$ __G__
   $x = \frac{y^2-2z^2-10}{5}$

2. $-x^2 + y^2 - 4z^2 = 12$ __C__
   $y^2 = 12 + x^2 + 4z^2$

3. $3x^2 + 3y^2 - 5z^2 = 0$ __E__
   $5z^2 = 3x^2 + 3y^2$

4. $x + 2(y + 3)^2 + 4z^2 = 28$ __D__
   $x = 28 - 2(y+3)^2 - 4z^2$

5. $-5x + y - 2z = 10$ __A__

6. $-x^2 - y^2 - 4z^2 = 12$ __H__
   $x^2 + y^2 + 4z^2 = -12$

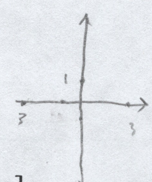
7. $3x^2 + 3y^2 - 5z^2 = 11$ __B__
   $z^2 = 3x^2 + 3y^2 - 11$

8. $x = z$ __A__

9. Consider the graph of $r = 3 - 2\sin\theta$. Circle **ALL** of the statements below that are true. [6]

**I.** It is a limaçon  **II.** It has a dimple  **III.** It's symmetric about the x axis

**IV.** It's symmetric about the y axis  **V.** It's max r – value is 5  **VI.** It has an inner loop



10. Which of the following is an equation of a rose curve with 10 petals? (circle 1 answer) [3 pts]

a) $r = 5\cos(10\theta)$  b) $r = 5\cos(5\theta)$  c) $r = 5\sin(5\theta)$

d) $r = 5\sin(10\theta)$  e) None of these

$\sin x = -\sin x$

cross-sections?

11. The traces of a hyperboloid of 2 sheets are: (circle 1 answer) [3]

a) two hyperbolas and one parabola  b) one hyperbola and two parabolas

c) two hyperbolas and one ellipse  d) one hyperbola and two ellipses

e) none of these

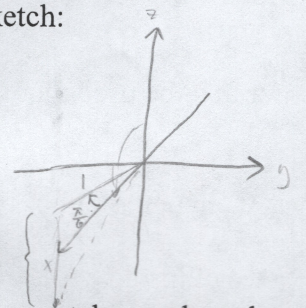
If traces means "traces," then it would be (e)

/-3

**Polar and 3D Free Response:**

12. Sketch the cylindrical point $(r,\theta,z) = \left(1,-\frac{\pi}{6},-1\right)$ and then convert it into spherical coordinates. [4]

Rough Sketch:



$\rho = \sqrt{1^2+1^2} = \sqrt{2}$

$\theta = -\frac{\pi}{6}$

$\phi = 135^\circ = \frac{3\pi}{4}$

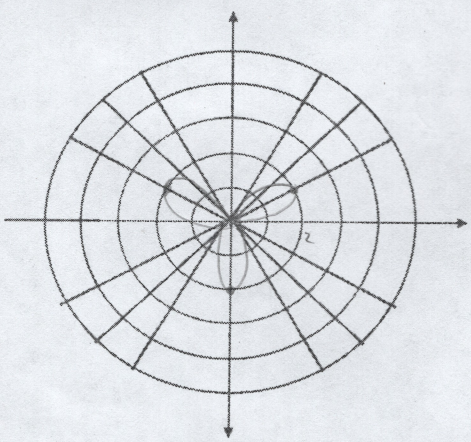
$(\rho,\theta,\phi)=$ $\left(\sqrt{2}, -\frac{\pi}{6}, \frac{3\pi}{4}\right)$

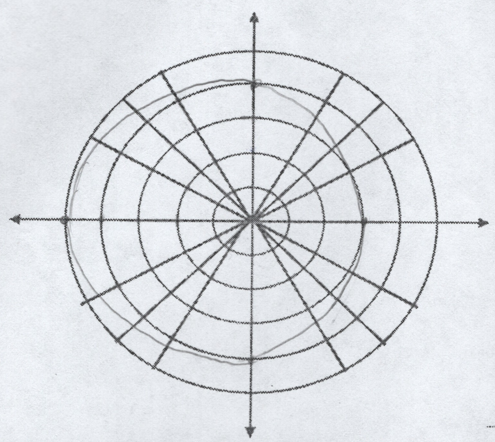
13. Quickly but accurately graph each polar curve below. [ 3 pts each]

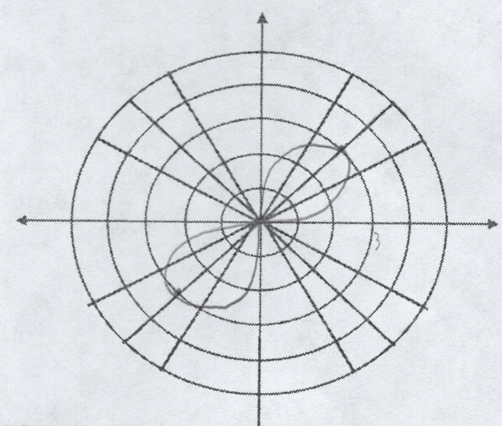
a) $r = 2\sin 3\theta$

b) $r = 4 - \cos\theta$

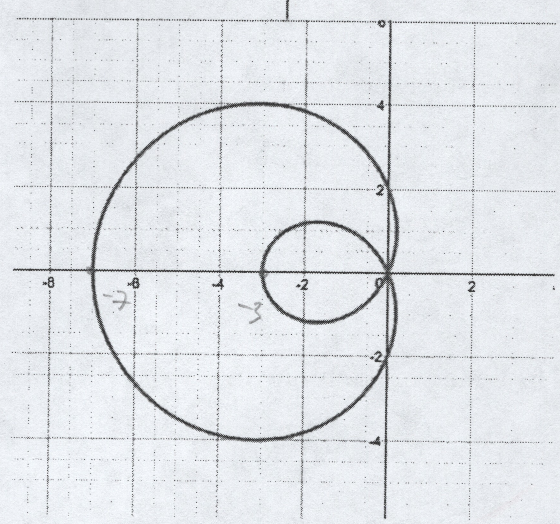
c) $r^2 = 9\sin 2\theta$







14. Write the equation of the graph on the right in polar form. [4]



$r$ = $2 - 5\cos\theta$

15. Convert $x^2 + y^2 = 2\sqrt{x^2+y^2} - 2y$ to **polar** form, and then identify the shape by its most specific name. [4]

$r = \sqrt{x^2+y^2}$ $r\sin\theta = y$ | $r\cos\theta = x$ $r \neq 0$

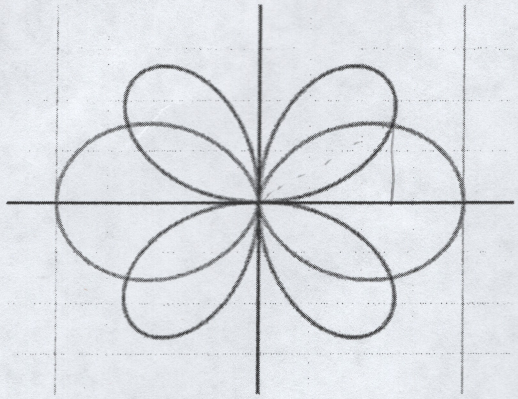
$r^2 = 2r - 2r\sin\theta \rightarrow r^2 = 2r(1-\sin\theta) \rightarrow r = 2 - 2\sin\theta$

Polar form: $r = 2 - 2\sin\theta$

Name: cardioid (special case of limaçon)

/10

16. The curves r = $2\cos^2\theta$ and r = $\sqrt{3}\sin 2\theta$ (graphed below) cross 5 times. Find two of the intersection points and write them in polar form. Show the algebra that leads to your answer for full credit. [5]



$r = 2\cos^2\theta \qquad r = \sqrt{3}\sin 2\theta$

$2\cos^2\theta = \sqrt{3}\sin 2\theta \qquad \cot 30° = \frac{\sqrt{3}/2}{1/2} = \sqrt{3}$

$\pi/6$

$\cos^2\theta = \sqrt{3}\cos\theta\sin\theta$ ) if $\cos\theta \neq 0$. If $\cos\theta = 0$,

$\cos\theta = \sqrt{3}\sin\theta$ then $0 = 0$ ✓ giving

$\theta = \cot^{-1}(\sqrt{3})$ is a solution $\quad r = 0,\ \theta = \frac{\pi}{2}$ (point 1)

$\theta = \frac{\pi}{6} \to r = \frac{3}{2}$ (point 2)

point 1 $(0, \frac{\pi}{2})$ point 2 $(\frac{3}{2}, \frac{\pi}{6})$

17. Any ellipsoid can be written in the form $\frac{(x-a)^2}{d} + \frac{(y-b)^2}{e} + \frac{(z-c)^2}{f} = 1$.

Create an ellipsoid that has its center at (1, 0, 0) and x-intercepts 4 and –2. Also make it have y-intercepts ±5, and pass through the point (3,0,1). [4]

$\frac{1}{9} + \frac{25}{x} = 1 \qquad \frac{25}{x} = \frac{8}{9}$

$8x = 9 \cdot 25$

$\frac{(x-1)^2}{3^2} + \frac{y^2}{(15\sqrt{2}/4)^2} + \frac{z^2}{9/5} = 1 \qquad 225 \qquad x = \left(\frac{15}{2\sqrt{2}}\right)^2 = \left(\frac{15\sqrt{2}}{4}\right)^2$

$\frac{4}{9} + \frac{1}{9 \cdot 5/5^2} = \frac{4}{9} + \frac{5}{9} = 1$

$$\boxed{\frac{(x-1)^2}{3^2} + \frac{y^2}{(15\sqrt{2}/4)^2} + \frac{z^2}{(3\sqrt{5}/5)^2} = 1}$$

$\frac{1}{9} + \frac{25}{9 \cdot 25 \cdot 2/16} = \frac{1}{9} + \frac{1}{9 \cdot \frac{1}{8}} = \frac{1}{9} + \frac{8}{9} = 1$

**PART 2: Vectors and Parametric Equations**

18. **Multiple Choice:** The graph of the set of parametric equations $\begin{aligned} x(t) &= \cos t \\ y(t) &= 3 - \sin^2 t \end{aligned}$ is a parabola. [3]

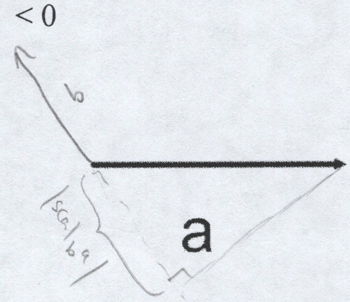
(graphed between $x = -1, 1$)

a) circle b) ellipse c) parabola d) hyperbola e) spiral

$\sin^2 t = 1 - \cos^2 t = 1 - x^2 \to y = 3 - (1 - x^2) = 2 + x^2$

19. Vector **a** is drawn below. Draw and label another vector **b** such that… [4]

a) ***a x b*** would have a direction **up** perpendicular out of this piece of paper.
b) The scalar projection $proj_b a < 0$




/ -0

20. Find the equation of the plane, in standard Ax+By+Cz+D = 0 form, that contains the following 3 non-collinear points: [6]

(2, 0, 3) (3, -1, 1) and (0, 4, 4)

$\langle 1,-1,-2\rangle, \langle -2,4,1\rangle$

$$\begin{vmatrix} i & j & k \\ 1 & -1 & -2 \\ -2 & 4 & 1 \end{vmatrix} = i\begin{vmatrix} -1 & -2 \\ 4 & 1 \end{vmatrix} - j\begin{vmatrix} 1 & -2 \\ -2 & 1 \end{vmatrix} + k\begin{vmatrix} 1 & -1 \\ -2 & 4 \end{vmatrix}$$

$$= i(-1+8) - j(1-4) + k(4-2)$$

$$= 7i + 3j + 2k$$

$$\boxed{7x+3y+2z-20=0}$$

$2,0,3 \rightarrow 14+6-20$ ✓

$3,-1,1 \rightarrow 21-3+2$ ✓

$0,4,4 \rightarrow 3\cdot 4+2\cdot 4-20$ ✓

21. Grayson is launching a grapefruit off the top of a building. The position (in feet) of the grapefruit after t seconds is given by the set of parametric equations:

$$x(t) = 40t\sqrt{3}$$
$$y(t) = 40t - 16t^2 + 70$$

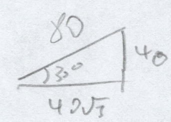
**Answer the following series of short answer questions about this scenario. [8]**

a) True or False: The grapefruit is launched from an initial height of 70 feet off the ground. T

$y(0) = 70$

b) The grapefruit was launched at a velocity of 80 f/s at an angle of 30 degrees

$y'(t) = -32t + 40, \; x'(t) = 40\sqrt{3} \rightarrow x'(0), y'(0) = (40\sqrt{3}, 40)$




c) After 1 second, the grapefruit is at a height of 94 feet.

$y(1) = 40 - 16 + 70 = 94$ $\quad 110 - 16 = 94$

d) The 2[nd] time the grapefruit will be 70 feet off the ground is at 2.5 seconds.

$y(t) = 70$

$40t - 16t^2 \cancel{+70} = \cancel{70}\, 0$

$\overset{100}{\cancel{40}}\left(\frac{5}{\cancel{2}}\right) - \overset{100}{\cancel{16}}\frac{25}{\cancel{4}} = 0$

$t(40-16t) = 0 \rightarrow t = \frac{40}{16} = \frac{10}{4} = \frac{5}{2}$

$\frac{}{}$ 20

22. Consider the two vectors **r** = <6, 8, 0> and **s** = <2, 2, -1> Fill in the blanks below either with <, >, = or NEI (not enough information) [2/2/2/2/4]

$6\cdot 2 + 8\cdot 2 = 12+16 = 28$ $6^2+8^2 = 100$

a) $\boldsymbol{r}\cdot\boldsymbol{s}$ __=__ 28 $2^2+2^2+1^2 = 9$

$r\cdot s = \|r\|\,\|s\| \cos\theta$

b) The angle between the two vectors __<__ 60 degrees

$\cos\theta = \frac{r\cdot s}{\|r\|\,\|s\|} = \frac{28}{30}$ $\cos 60° = \frac{\sqrt{3}}{2}$

c) $scalar\ proj_s r$ __>__ $|s|$

$\frac{r\cdot s}{\|s\|^2}\cdot s = \frac{28}{9}\cdot\langle 2,2,-1\rangle$

d) The area of the parallelogram formed by the two vectors __>__ 10 square units

e) Now, using the same vectors **r** and **s**, calculate the distance from the point (-3, -2, 1) to the plane formed by vectors **r**, **s** and the origin.

$\begin{vmatrix} i & j & k \\ 6 & 8 & 0 \\ 2 & 2 & -1 \end{vmatrix} = i\begin{vmatrix} 8 & 0 \\ 2 & -1 \end{vmatrix} - j\begin{vmatrix} 6 & 0 \\ 2 & -1 \end{vmatrix} + k\begin{vmatrix} 6 & 8 \\ 2 & 2 \end{vmatrix} = i(-8) - j(-6) + k(12-16) = -8i+6j-4k$

$\to \sqrt{4^2+6^2+8^2} = 2\sqrt{2^2+3^2+4^2} = 2\sqrt{29} > 2\sqrt{25} = 10$

$-8x+6y-4z=0$

$\downarrow$

$8x-6y+4z=0$

$dist = \frac{-24+12+4}{2\sqrt{29}} = \frac{-8}{2\sqrt{29}} = -\frac{4}{\sqrt{29}}$

(-1) $\boxed{+\frac{4}{\sqrt{29}}}$

23. Name a plane (in standard form) perpendicular to the plane 3x – 5y +2z = 20. Then using words and math, convince me that your answer is correct. Many answers are possible. [4]

$\langle 3,-5,2\rangle \cdot \langle 5,3,0\rangle = 15-15+0 = 0$

Your plane $5x+3y=1$

Your argument: Two planes are ⊥ iff their normal vectors are also ⊥.

A normal of the given plane is <3,-5,2>, and a normal of my plane is <5,3,0>. Since <3,-5,2>·<5,3,0> = 15-15+0 = 0, the two vectors are perpendicular, and thus the planes are perpendicular.

-1

24. Consider line L: $< x,y,z >=< -2,5,1 > + < 1,2,-4 > t$

a) Is the point (98, 205, -350) on line L? Justify your answer. [3]

$x = -2 + t$
$y = 5 + 2t \longrightarrow t = 100$
$z = 1 - 4t$

Yes or no: no

Justification: When $t = 100$, we have $x = -2 + t = 98$ and $y = 5 + 2t = 205$, which matches the x and y coords of the point. The z value, however, is $z = 1 - 4t = -399 \neq -350$, so the point is not on the line. We can do this because each x value corresponds with one value of t, so the only t value that might contain the point (by the criterion of = x values) is $t = 100$, which fails.

b) Line L above intersects this new line $< x,y,z >=< 2,-2,27 > + < 3,1,2 > t$
Find the point of intersection of the two lines. [3]

$x = 2 + 3t$
$y = -2 + t$
$z = 27 + 2t$

$2 + 3t_2 = -2 + t_1 \rightarrow 4 + 6t_2 = -4 + 2t_1$
$5 + 2t_1 = -2 + t_2 \rightarrow -2 + t_2 = 5 + 2t_1$
$1 - 4t_1 = 27 + 2t_2 \quad 6 + 5t_2 = -9 + 0$

$5t_2 = -15$
$t_2 = -3$
$t_1 = -5$

$x = 2 + 3(-3) = -7$

oops!

(x,y,z) = ~~(-7, -5, 21)~~

1 - 0