Group Theory Quiz 1
Analysis 2018-19 [33 points]

Herreshoff Groupie Timothy H Period: 6

28/28 wow

1. Consider the four-member collection of elements A, B, C, D as displayed in the table below under the operation $.

$	A	B	C	D
A	C	D	B	A
B	D	C	A	B
C	B	A	D	C
D	A	B	C	D

a) Is there an identity element in this group? If so, name it and defend your answer. If not, justify. [2]

D is an identity element, because D$X = X$D = X for all X. On the table, this manifests itself as the row "ABCD" and column "ABCD" for D.

b) Does every element have an inverse element? Name them and justify your answer. [3]

A's inverse is B and vice versa; C's inverse is itself, and D's inverse is itself

B$A = A$B = D = identity, C$C = D, D$D = D

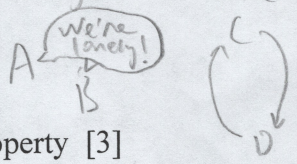
c) Name the **period** of element B (or state that it does not have one) 4 [2]

(B$B)$(B$B) = (C)$(C) = D = identity

d) Is the element C (by itself) a generator of the "group"? How do you know? [3]

No. When repeatedly applying $ to C, only elements C and D are generated. Since A, B are not generated, C is not a generator

C = C
C$C = D
CCC = C looped around



e) Does this collection of elements satisfy the commutative property [3]

Yes. This property is shown by the fact that it is symmetric across the TL-BR diagonal. For all X, Y, we have X$Y ∈ G, where G is the group.

2. Consider the silly "12 post snap group" How many different elements would there be? 12! Would you like to create a table for this group? Hell no! How many different entries would be in such a table? $12!^2$ [3 total]

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3. Does the set of numbers {1, 3, 1/3, -3, -1/3} form a group under multiplication? Justify your answer mathematically. Let S = the given set.

No. Since $\frac{1}{3}, -\frac{1}{3} \in S$, but $\frac{1}{3} \cdot -\frac{1}{3} = -\frac{1}{9} \notin S$, the "group" does not exhibit closure and is therefore not a group.

4. Fill in the blanks.

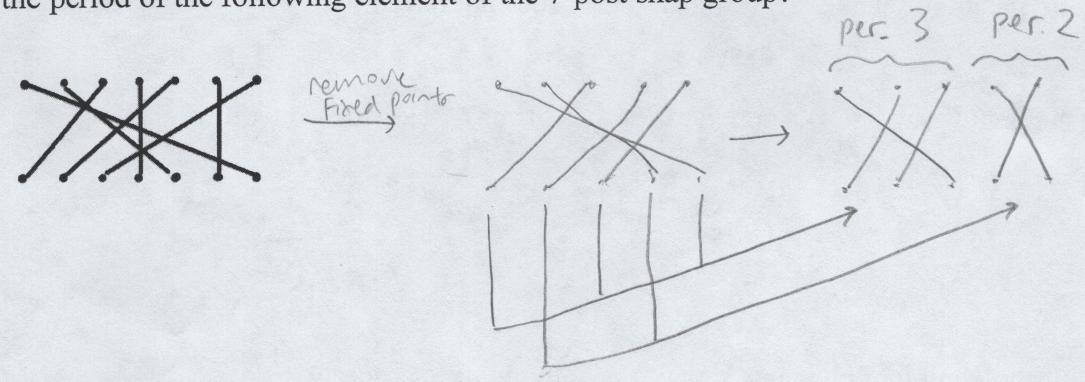
a) The rotation group of a regular hexagon would have 6 elements

b) The reflection group of a non-equilateral, but isosceles triangle has 2 elements.

c) The group formed by the two operations "rotate 20 degrees" and "reflect over the x axis" would have 36 elements.

$\frac{360}{20} \cdot 2 = 18 \cdot 2 = 36$

5. What is the period of the following element of the 7-post snap group?



lcm (3, 2) = 6

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