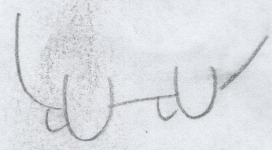
Analysis H – Deggeller / Gleason / Hahn
Limits of Sequences and Series, Quiz 1
No Calculators!

See Jack, see Jane, sequence. Now see Timothy Vlenke

Period: 6

Score: 20 / 20



1. Given the sequence: $a_n = \left\{\frac{3n^2+2}{n^2}\right\}$

 a) [2 pt] The sequence converges to 3

 b) [2 pts] Finish the sentence: We can show the convergence of part (a) above because for all neighborhoods, no matter how small, we can find a natural number M value such that...

 for all $n \geq M$, a_n is in the neighborhood of 3

 c) [3 pts] If the neighborhood has a value of E = 0.1, find the natural number value of M from part (b).

 $3 - E \leq \frac{3n^2+2}{n^2} \leq 3+E$

 $2.9 \leq \frac{3n^2+2}{n^2} \leq 3.1$

 $2.9n^2 \leq 3n^2+2 \leq 3.1n^2$ $[n > 0]$

 $\underbrace{-0.1n^2 \leq 2}_{\text{always true}} \leq .1n^2 \longrightarrow 20 \leq n^2 \rightarrow n \geq 5$, $\boxed{M=5}$

2. Tell whether each statement is True or False. [1 pts each]

 a) If a sequence does not converge, it must diverge. T

 b) If a sequence is bounded above and below, it must converge. F

 c) If a sequence is bounded below and everywhere decreasing, it must converge. T

 d) If it can be shown that for n > 8, all the terms of a_n are greater than n, the sequence $\{a_n\}$ must diverge. T

 e) ALL sequences that converge are bounded below. T

3. Given the sequence: $a_n = \left\{\frac{2n}{n+1}\right\}$

 a) [3 pts] Show that the sequence is bounded above.

 given $n \geq 1$, since n is an index

 $\underbrace{2n \leq 2n+2}_{\text{always true}} \rightarrow 2n \leq 2(n+1) \longrightarrow \boxed{\frac{2n}{n+1} \leq 2}$

 [factorins] [Since $n+1 > 0$]

 2 is an upper bound

 b) [3 pts] Show that the sequence is everywhere increasing.

 $\frac{2(n+1)}{(n+1)+1} - \frac{2n}{n+1} = \frac{2(n+1)^2 - 2n(n+2)}{(n+2)(n+1)} = \frac{2n^2+4n+2-2n^2-4n}{(n+2)(n+1)}$

 $= \frac{2}{(n+2)(n+1)} > 0$ [Since $n+2, n+1 > 0$]

 So $\frac{2(n+1)}{(n+1)+1} - \frac{2n}{n+1} > 0$

 $\Rightarrow a_{n+1} - a_n > 0$

 $\Rightarrow \boxed{a_{n+1} > a_n}$

 c) [2 pts] What can you conclude from parts (a) and (b) together?

 a_n converges