Analysis Honors – Deggeller / Gleason / Hahn   I am vectorious!: Timothy Herhen
Unit 4: Vectors and Parametrics, Quiz 1   [30]   Period: 6
No Calculators, write answers in exact form.   2018-19 $\frac{30}{30}$

1. Consider the cylindrical point P= $(r, \theta, z) = (5, 30^o, 3)$. Convert P to both rectangular and spherical.

Rectangular (x, y, z) = $\left(\frac{5\sqrt{3}}{2}, \frac{5}{2}, 3\right)$ [3]

$(x, y) = (5\cos 30^\circ, 5\sin 30^\circ)$

$\frac{5\sqrt{3}}{2}$ $\frac{5}{2}$

toa

$\tan\theta = \frac{5}{3}$

Spherical (ρ, θ, ϕ)= $\left(\sqrt{34}, 30^\circ, \tan^{-1}\left(\frac{5}{3}\right)\right)$ [3]

$25 + 9 = 34$

$\sqrt{25+9} = \sqrt{34}$

$\tan^{-1}x + \tan^{-1}\left(\frac{1}{x}\right) = \frac{\pi}{2}$

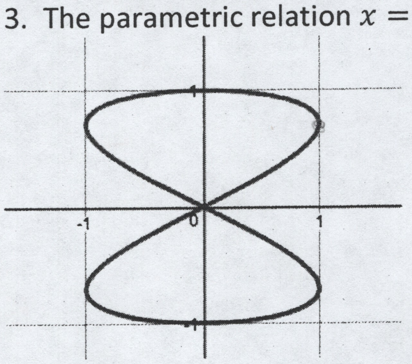
2. Eliminate the parameter for the parametric equation: $y = 3t - 1$ and x= $\frac{3}{t+5}$, simplify, and identify the shape. [2]

$y = 3t - 1, \quad x = \frac{3}{t+5}$

$t = \frac{y+1}{3} \longrightarrow x = \frac{3}{\frac{y+1}{3} + 5} \cdot \frac{3}{3} = \frac{9}{y+1+15} = \frac{9}{y+16}$

Function $x = f(y) = \frac{9}{y+16}$ shape hyperbola

3. The parametric relation $x = sin2t$ and $y= sint$ is graphed below over the interval $[0,2\pi]$



max of $\sin 2t \rightarrow 2t = 90^\circ = \frac{\pi}{2}$

$t = \frac{\pi}{4}$

a) Name a t value when the graph is furthest to the right t= $\frac{\pi}{4}$ [2]

b) Eliminate the parameter to form a relationship in x, and y without trig functions. [2]

Test: $t = \frac{\pi}{4} \rightarrow (x, y) = \left(1, \frac{\sqrt{2}}{2}\right)$

$\frac{1}{2}$

$x = \sin 2t \quad y = \sin t$

$x = 2\cos t \sin t \quad y = \sin t$

$x^2 = 4\cos^2 t \sin^2 t \quad y = \sin t$

$x^2 = 4(1 - \sin^2 t)\sin^2 t \rightarrow \boxed{x^2 = 4(1 - y^2)y^2}$

$x^2 = 4\left(1 - \frac{1}{2}\right)\frac{1}{2}$

c) Explain what would happen to the graph if the t range were expanded to [-100, 100] [1]

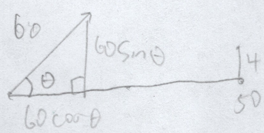
The graph is periodic, (with periods π, 2π for x, y respectively), so it would look the same; it'd just get traced over around $\frac{100}{\pi}$ times.

-0

4. Mr. Redfield is chipping a golf ball off the ground with initial velocity 60 ft/sec. He is trying to determine the angle to chip the ball so that in 50 horizontal feet the ball will land 4 feet above his current elevation. Find a **single equation with one variable "theta" in it** that could be solved (using a grapher or solver) to help Mr. Redfield save par. Again, you don't need to solve the equation (but if you have time give it a whack). [4]

$y = (60\sin\theta)t - 16t^2$

$x = (60\cos\theta)t \rightarrow t = \frac{x}{60\cos\theta}$



$x = 50,\ y = 4$

$\rightarrow y = \frac{60\sin\theta}{60\cos\theta}\cdot x - 16\left(\frac{x}{60\cos\theta}\right)^2$

$= x\tan\theta - 16\left(\frac{x}{60\cos\theta}\right)^2$

I assume we need not simplify

Equation with one variable theta: $4 = (60\sin\theta)\left(\frac{50}{60\cos\theta}\right) - 16\left(\frac{50}{60\cos\theta}\right)^2$

5. Consider the points A= (-3, 5) and B = (4,6) and the vectors $\vec{r} = \langle 3,6 \rangle$ and $\vec{s} = \langle -5,4 \rangle$

a) Find vector $2\overrightarrow{BA}$ $\langle -14, -2 \rangle$ [2] $\langle -7, -1 \rangle$

b) $|\vec{s}|$ = $\sqrt{41}$ [2] $\sqrt{25+16} = \sqrt{41}$ $\sqrt{25+16} \rightarrow \sqrt{41}$

c) $\vec{r}\cdot\vec{s}$ = 9 [2] $3 \cdot -5 + 6\cdot 4 = -15 + 24 = 9$ $24 - 15 = 9$

d) The angle between $\vec{r}$ *and* $\vec{s}$ $\cos^{-1}\left(\frac{3}{\sqrt{205}}\right)$ [2]

$\theta = \cos^{-1}\left(\frac{9}{\sqrt{41}\cdot\sqrt{45}}\right)$

$= \cos^{-1}\left(\frac{3}{\sqrt{205}}\right)$

$\sqrt{3^2+6^2} = \sqrt{45}$ $\sqrt{5^2+4^2} = \sqrt{25+16} = \sqrt{41}$

$= 3\sqrt{5}$

$v\cdot v = |v||v|\cos\theta$

$\Rightarrow 9 = \sqrt{45}\cdot\sqrt{41}\cdot\cos\theta \Rightarrow 9 = 3\sqrt{5}\sqrt{41}\cos\theta$

$\Rightarrow 3 = \sqrt{205}\cos\theta$

$\theta = \cos^{-1}\left(\frac{3}{\sqrt{205}}\right)$

e) The point 2/3 of the way from A to B. $\left(\frac{5}{3}, \frac{17}{3}\right)$ [2]

$-3 + \frac{14}{3},\ 5 + \frac{2}{3}$

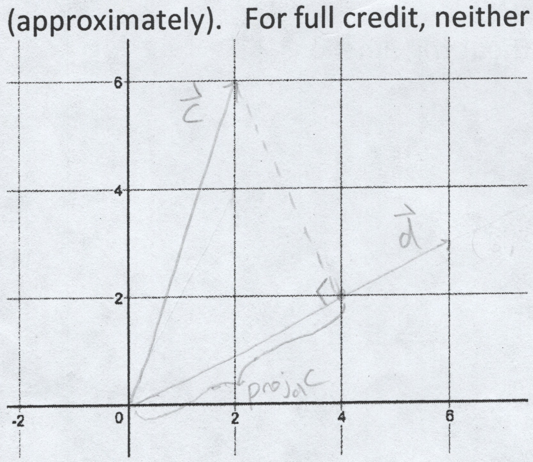
$= \frac{5}{3},\ \frac{17}{3}$

$\langle -3, 5 \rangle + \frac{2}{3}\langle 7, 1 \rangle = \langle -3 + \frac{14}{3}, 5 + \frac{2}{3} \rangle$

$= \langle -\frac{9}{3} + \frac{14}{3}, \frac{15}{3} + \frac{2}{3} \rangle$

$= \langle \frac{5}{3}, \frac{17}{3} \rangle$

6. Sketch AND LABEL two vectors $\vec{c}$ *and* $\vec{d}$ below such that the vector projection $proj_d c$= <4, 2> (approximately). For full credit, neither $\vec{c}$ *nor* $\vec{d}$ should be <4,2> (because that's boring) [3]



$\vec{c} = \langle 2, 6 \rangle \rightarrow 40$

$\vec{d} = \langle 6, 3 \rangle \rightarrow 45$

$40 \cdot 45 = 1800$

$d\cdot\frac{c\cdot d}{|d|^2} = \frac{12+18}{45}\cdot\langle 6,3 \rangle$

$= \frac{30}{45}\cdot\langle 6,3 \rangle$

$= \frac{2}{3}\cdot\langle 6,3 \rangle$

$= \boxed{\langle 4,2 \rangle}$

✓ 0