Analysis H – Deggeller / Hahn/Gleason 18-19   Welcome to Pascalifornia! Timothy Hirahara
Unit 1, Quiz 1 – ATPS [30 pts]   Period: 6
NO CALCULATORS, but feel free to leave answers unsimplified.

30/30

1. Consider the odd number triangle we studied in class, starting with row 1.

$\frac{2(2+1)}{2} = 3$

			1			
		3		5		
	7		9		11	
13		15		17		19

a) Name the middle term of the 55th row. $55^2 = 3025$ [2]

55×55: 275, 2750, 3025

b) **How many** terms (total) are in the first 9 rows of the triangle? 45 [3]

$\frac{9(9+1)}{2} = \frac{90}{2} = 45$

c) In class we proved that the first term of the nth row is $n^2 - n + 1$. Knowing this, find an expression for the last term of the nth row. [3]

$n^2 + 2n + 1 - n - 1 + 1 - 2 = n^2 + n - 1$

$(n+1)^2 - (n+1) + 1 - 2$

$= n^2 + 2n + 1 - n - 1 + 1 - 2$

$= \boxed{n^2 + n - 1}$

2. Fill in the blanks. [3 each]

a) $F_{25} = \underline{8}\, F_{20} + \underline{5}\, F_{19}$

$5F_{21} + 3F_{20} \to 3F_{22} + 2F_{21} \to 2F_{23} + 1F_{22}$
$\to 1F_{24} + 1F_{23} \to F_{25}$

$F_{25} = F_{24} + F_{23} = F_{23} + F_{22} + F_{22} + F_{21}$ $3F_{21} + 5F_{20} + 2F_{19}$

$= 3F_{22} + 2F_{21}$

$= 3(F_{21} + F_{20}) + 2(F_{20} + F_{19})$

b) $F_{217} = F_{\underline{218}} - F_{\underline{216}}$ *or* $F_{\underline{219}} - F_{\underline{218}}$

F_{216} F_{217} F_{218} F_{219}

−0

3. Briefly explain the relationship between the Fibonacci Numbers and the Golden Ratio using words, mathematical symbols and/or pictures. [3]

The ratio between consecutive Fibonacci numbers approaches ϕ $\left(\frac{1+\sqrt{5}}{2}\right)$. In other words, $\lim_{n\to\infty}\frac{F_n}{F_{n-1}}=\phi$

4. Evaluate each, leaving your answer in terms of choose numbers or whole numbers [3 each]

a) $\binom{142}{35}+\binom{142}{36}=$ $\binom{143}{36}$

$\binom{4}{2}+\binom{4}{3}=6+4=10$

$\binom{5}{2}+\binom{5}{3}=\binom{6}{3}$

$\frac{5\cdot 4\cdot 3}{3\cdot 2}=10$

b) $\binom{n}{1}+\binom{n}{3}+\binom{n}{5}+\ldots+\binom{n}{n}=$ 2^{n-1}

(assume n is an odd number bigger than 5)

$\binom{3}{1}+\binom{3}{3}=3+1=4$

$2^3=8$

$\binom{3}{3}+\binom{4}{3}+\binom{5}{3}$

$\binom{6}{4}$

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1

c) $\binom{42}{40}+\binom{43}{40}+\binom{44}{40}+\ldots\ldots\binom{104}{40}=$ $\binom{105}{41}-42$

$\binom{40}{40}+\cdots+\binom{104}{40}=\binom{105}{41}$

$\binom{41}{40}+\binom{40}{40}=41+1=42$

5. **Choose 1 of the following 2 problems to do. Circle the problem you want me to grade.** [4]

a) Find a compact (simplified) expression for the sum of the first n Fibonacci numbers.

OR

b) There is a family of numbers called pentagonal numbers, the first 6 of which are:

1, 5, 12, 22, 35, 51……. Find an expression for the nth pentagonal number.

0 1 1 2 3 5 8 13 21

F_0 F_1 F_2 F_3 F_4 F_5 F_6 F_7 F_8

F_0: 1 0 1 1 2 3 5 8

F_1: 0 1 1 2 3 5 8 13

0, 1, 1, 2, 3, 5, 8, 13, 21, 34

0, 1, 2, 4, 7, 12, 20, 33, 54

33

$F_2+F_4+F_6+F_8=F_9-1$

$$\boxed{\text{So } \sum_{i=0}^{n}F_i=F_{n+2}-1}$$

$F_2+F_4+F_6+\cdots+F_n=F_{n+1}-1$

Ex: 7 $(F_1=1,\ F_2=1)$

$F_1+F_2+\cdots+F_7=33$

$F_9-1=34-1=33$