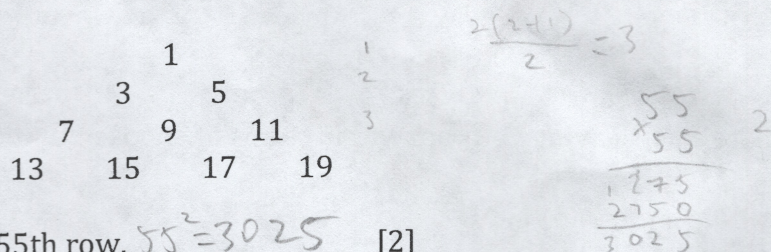


30
30

1. Consider the odd number triangle we studied in class, starting with row 1.



a) Name the middle term of the 55th row. $55^2 = 3025$ [2]

b) **How many** terms (total) are in the first 9 rows of the triangle? 45 [3]

$$\frac{9(9+1)}{2} = \frac{90}{2} = 45$$

c) In class we proved that the first term of the n th row is $n^2 - n + 1$. Knowing this, find an expression for the last term of the n th row. [3]

$$\begin{aligned} n^2 + 2n + 1 - n - 1 + 1 - 2 &= n^2 + n - 1 \\ (n+1)^2 - (n+1) + 1 - 2 \\ &= \frac{n^2 + 2n + 1}{1} - n - 1 + 1 - 2 \\ &= \boxed{n^2 + n - 1} \end{aligned}$$

2. Fill in the blanks. [3 each]

a) $F_{25} = \underline{8} F_{20} + \underline{5} F_{19}$

$$\begin{aligned} 5F_{21} + 3F_{20} &\rightarrow 3F_{22} + 2F_{21} \rightarrow 2F_{23} + 1F_{22} \\ &\rightarrow 1F_{24} + 1F_{23} \rightarrow F_{25} \end{aligned}$$

$$\begin{aligned} F_{25} &= F_{24} + F_{23} = F_{23} + F_{22} + F_{22} + F_{21} \\ &= 3F_{22} + 2F_{21} \\ &= 3(F_{22} + F_{20}) + 2(F_{20} + F_{19}) \end{aligned}$$

b) $F_{217} = F_{\underline{218}} - F_{\underline{216}} \text{ or } F_{\underline{219}} - F_{\underline{218}}$

$$F_{216} \quad F_{217} \quad F_{218} \quad F_{219}$$

- 0

3. Briefly explain the relationship between the Fibonacci Numbers and the Golden Ratio using words, mathematical symbols and/or pictures. [3]

The ratio between consecutive Fibonacci numbers approaches $\phi = \frac{1+\sqrt{5}}{2}$. In other words,

$$\lim_{n \rightarrow \infty} \frac{F_n}{F_{n-1}} = \phi$$

4. Evaluate each, leaving your answer in terms of choose numbers or whole numbers [3 each]

a) $\binom{142}{35} + \binom{142}{36} = \underline{\binom{143}{36}}$

$\binom{4}{2} + \binom{4}{3} = 6 + 4 = 10$

$\binom{5}{2} + \binom{5}{3} = \binom{6}{3}$

$\frac{5 \cdot 4 \cdot 3}{3 \cdot 2} = 10$

b) $\binom{n}{1} + \binom{n}{3} + \binom{n}{5} + \dots + \binom{n}{n} = \underline{2^{n-1}}$

$\binom{3}{1} + \binom{3}{3} = 3 + 1 = 4$

$2^3 = 8$

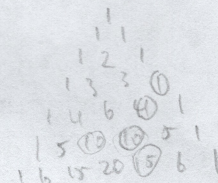
$\binom{3}{3} + \binom{4}{3} + \binom{5}{3}$

(assume n is an odd number bigger than 5)

c) $\binom{42}{40} + \binom{43}{40} + \binom{44}{40} + \dots + \binom{104}{40} = \underline{\binom{105}{41} - 42}$

$\binom{40}{40} + \dots + \binom{104}{40} = \binom{105}{41}$

$\binom{41}{40} + \binom{40}{40} = 41 + 1 = 42$



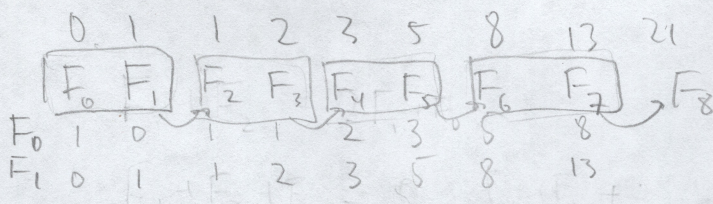
5. Choose 1 of the following 2 problems to do. Circle the problem you want me to grade. [4]

a) Find a compact (simplified) expression for the sum of the first n Fibonacci numbers.

OR

b) There is a family of numbers called pentagonal numbers, the first 6 of which are:

1, 5, 12, 22, 35, 51..... Find an expression for the nth pentagonal number.



0	0
1	1
1	2
2	4
3	7
5	12
8	20
13	33
21	54
34	

33

$F_2 + F_4 + F_6 + F_8 = F_9 - 1$

So $\sum_{i=0}^n F_i = F_{n+2} - 1$

$F_2 + F_4 + F_6 + \dots + F_n = F_{n+1} - 1$

Ex: 7

$(F_1 = 1, F_2 = 1)$

$F_1 + F_2 + \dots + F_7 = 33$

$F_9 - 1 = 34 - 1 = 33$

(Handwritten red mark: a large slash followed by a minus sign and a zero)