

Calculators OK

1. The sum of a geometric series can be computed using the formula  $S = \frac{a(1-r^n)}{1-r}$

a) A first step in deriving the formula is to begin with a generic geometric series

$$S = a + ar + ar^2 + ar^3 + \dots + ar^{n-1}$$

And multiply both sides by  $r$ :

$$Sr = ar + ar^2 + ar^3 + ar^4 + \dots + ar^n$$

Using the two equations above, complete the derivation for the sum formula. Obviously, you already know the formula, so your answer will be graded on the clarity of your steps.

b) Find the sum of the following geometric series:

$$6 - 9 + \frac{27}{2} - \frac{81}{4} + \dots + \frac{129140163}{32768}$$

c) Use mathematical induction to prove that the geometric sum formula works for all positive  $n$ .

$$\text{Prove: } a + ar + ar^2 + ar^3 + \dots + ar^{n-1} = \frac{a(1-r^n)}{1-r}$$



2. Find the sum of  $1 - 2 + 3 - 4 + 5 - \dots + 497 - 498 + 499 - 500$ . Show your work!

3. Simplify completely (you can leave your answers in factored form, but your final expressions should not have factorials in them).

a)  $\frac{(n^2-4)!}{(n-2)(n^2-5)!}$

b)  $\frac{(n!)^2}{(n-1)!(n+1)!}$       b) es shossior

4. The sum of the first 62 terms of a certain arithmetic series is 15,934. If the common difference of the sequence is 8, what is the first term of the series?

5. Most mathematicians would say that the series below "diverges" and has no sum, because the sum goes to positive/negative infinity. However, if you HAD to give it a sum based on the patterns we've found in Pascal's Triangle, what would be the most appropriate value?

$$1 - 5 + 15 - 35 + 70 - 126 + 210 - 330 + 495 - \dots = \underline{\hspace{2cm}}$$