Analysis Limits Quiz2 ______________says, "This quiz is as easy as 1, 1, 2, 3..."
[30 pts]

1. Find the limit of the given sequence or say that it diverges. No need to justify. [2 ea]

a) $a_n = \dfrac{2\sin^2 n + 2\cos^2 n}{5}$

b) $b_n = \left(5 + \dfrac{2}{n^2}\right)^4 \left(\dfrac{4n+3}{n}\right)$

c) $b_n = \left\{\dfrac{3\cos(n)}{n}\right\}$

2. Are the following statements Always, Sometimes, or Never true? [1 ea]

a). If a sequence is always increaseing, then it diverges. ________________

b) If a series $\sum a_n$ converges, then the sequence a_n converges to 0. ________________

c) An infinite series is convergent if the sequence of its partial sums is convergent. __________

d) If a sequence a_n converges to 0, then the series $\sum a_n$ will converge. ________________

3. Given $P_n = \dfrac{1}{3}, \dfrac{1}{15}, \dfrac{1}{35}, \ldots \dfrac{1}{4n^2 - 1}$

a) Determine whether P_n converges or diverges. Justify your answer using one of the tests we learned in class. [2]

b) Determine whether $\sum_{n=1}^{\infty} P_n$ converges or diverges. Justify your answer using one of the tests we learned in class. [2]

4. [4] Use one of the tests from class to determine convergence/divergence for

$\{Hn\} = \left\{1 - \dfrac{1}{2} + \dfrac{1}{4} - \dfrac{1}{8} + \dfrac{1}{16} \ldots\right\}$ State the name of the test used, and justify its usage.

5. [4] Use the Ratio test to determine convergence of $\sum_{n=0}^{\infty} \frac{n!}{5^n}$

6. Consider the sequence $t_n = \frac{-n}{2+3n}$, starting with n=1.

a) [1] The limit of this sequence (as n goes to infinity) is ______________

b) [2] Is the sequence always increasing, always decreasing, or neither? Justify your answer using algebra.

c) [2] Using your answer from b (and some more logic) prove that the sequence converges. Write a conclusion statement showing why your proof is valid.

d) [3] Prove that your answer to part "a" is right by showing that after a certain term (M), the sequence will be within a neighborhood of radius $\varepsilon = 1/30$ from your limit. Write a sentence at the end summarizing your logic.