Analysis H 2018-2019 – Deggeller / Gleason / Hahn

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Unit 4: Vectors and Parametrics, Quiz 2

Period: 6

Calculator OK

Score: 27 / 30 pts

1. Consider the vectors $\vec{u} = \langle 10,12,7\rangle$ and $\vec{v} = \langle -2,5,11\rangle$, find...

a) $\vec{u} \times \vec{v}$ [4 pts]

$$\begin{vmatrix} i & j & k \\ 10 & 12 & 7 \\ -2 & 5 & 11 \end{vmatrix} = i\begin{vmatrix} 12 & 7 \\ 5 & 11 \end{vmatrix} - j\begin{vmatrix} 10 & 7 \\ -2 & 11 \end{vmatrix} + k\begin{vmatrix} 10 & 12 \\ -2 & 5 \end{vmatrix} = i(12\cdot 11 - 5\cdot 7) - j(10\cdot 11 - (-2)\cdot 7) + k(10\cdot 5 - (-2)\cdot 12)$$

$$= i(97) - j(124) + k(74) = \boxed{\langle 97, -124, 74\rangle}$$

b) The equation of a plane, in parametric form, that contains the point (4, 0, 1) and the two vectors above.

[4 pts]

$\cancel{\langle x,y,z\rangle = \langle 4,0,1\rangle + \langle 10,12,7\rangle s + \langle -2,5,11\rangle t}$

$= \langle 4,0,1\rangle + \langle 6,12,6\rangle s + \langle -6, +5, +10\rangle t$

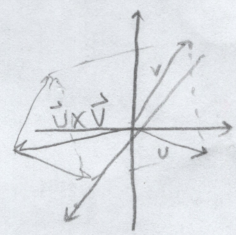
x = $4 + 6s + 6t$

y = $12s + 5t$

z = $1 + 6s + 10t$ -2

c) The volume of the triangular prism that has $\vec{u}$, $\vec{v}$, and $\vec{u} \times \vec{v}$ as 3 of its edges. Include a diagram in your work.
[4 pts]

diagram:



volume: $\frac{30261}{2}$ unit^3

$\frac{|\vec{u}\times\vec{v}|}{2}\cdot|\vec{u}\times\vec{v}|$ (bh)

2. A certain plane contains the point (3, -1, 2), and the line <x, y, z> = <0, 5, -2> + t<-6, 4, 1>. Write the equation of the plane in Ax + By + Cz = D form. [4 pts]

~~normal: $\langle -6, 4, 1\rangle$~~

~~$-6x + 4y + z = d$~~

~~$-6(3) + 4(-1) + 2 = -20 \rightarrow 6x - 4y - z = 20$~~

$\langle 3,-1,2\rangle$, $\langle 0,5,-2\rangle$, $\langle -6,9,-1\rangle$

A B C

$\vec{AB} = \langle -3, 6, -4\rangle$

$\vec{AC} = \langle -9, 10, -3\rangle$

$\boxed{22x + 27y + 24z = 87}$

$$\begin{vmatrix} i & j & k \\ -3 & 6 & -4 \\ -9 & 10 & -3 \end{vmatrix} = i\begin{vmatrix} 6 & -4 \\ 10 & -3 \end{vmatrix} - j\begin{vmatrix} -3 & -4 \\ -9 & -3 \end{vmatrix} + k\begin{vmatrix} -3 & 6 \\ -9 & 10 \end{vmatrix}$$

$= i(22) + 27(j) + 24k$

~~$\begin{vmatrix} i & j & k \\ -3 & 6 & -4 \\ -5 & 5 & -1 \end{vmatrix} = i\begin{vmatrix} 6 & -4 \\ 5 & -1 \end{vmatrix} - j\begin{vmatrix} -3 & -4 \\ -5 & -1 \end{vmatrix} + k\begin{vmatrix} -3 & 6 \\ -5 & 5 \end{vmatrix}$~~

~~$= i(-6+20) - j(3-36) + k(-15+54)$~~

~~$= 14i + 33j + 39k$~~

-2

3. Find the distance between the planes 2x + 3y + 4z = 8 and 2x + 3y + 4z =10. [3 pts]

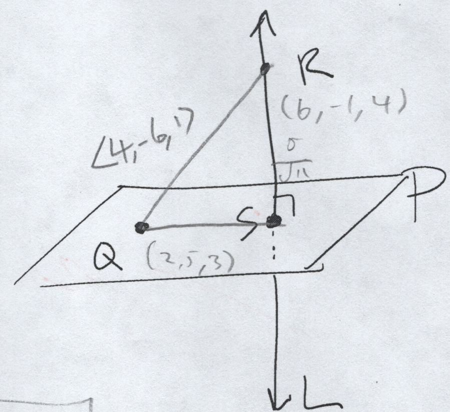
$2x+3y+4z=8$ $\quad$ $2x+3y+4z-10=0$

$p=(4,0,0)$

$$\frac{|2(4)-10|}{\sqrt{2^2+3^2+4^2}} = \frac{|-2|}{\sqrt{29}} = \boxed{\frac{2}{\sqrt{29}}}$$

4. Consider the plane P: 3x + y – z = 8, the point Q = (2, 5, 3), and the point R = (6, -1, 4). Q is on the plane, and R is not. Line L is normal to plane P and contains point R. Line L and plane P intersect at point S. [7 pts 2/2/3]

a) $|\overrightarrow{RS}|=$ $\dfrac{|3(6)+(-1)-4-8|}{\sqrt{3^2+1^2+1^2}} = \boxed{\dfrac{5}{\sqrt{11}}}$



b) $|\overrightarrow{QS}| = \sqrt{|\overrightarrow{QR}|^2 - |\overrightarrow{RS}|^2} = \sqrt{|\langle 4,-6,1\rangle|^2 - \left|\frac{5}{\sqrt{11}}\right|^2}$

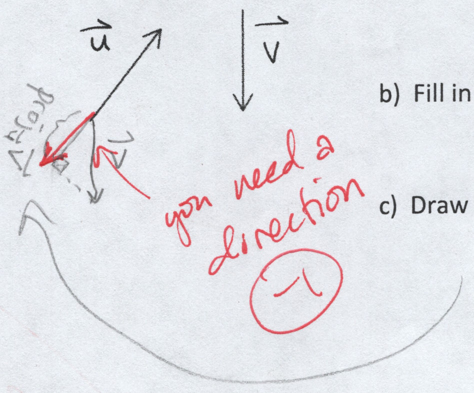
$$= \sqrt{4^2+6^2+1^2-\frac{25}{11}} = \sqrt{\frac{558}{11}} = \boxed{3\sqrt{\frac{62}{11}}}$$

c) coordinates of point T, such that $\overrightarrow{QT}$ (cutie!!) is a unit vector, in the opposite direction of $\overrightarrow{QR}$.

$$\overrightarrow{QR} = \langle 4,-6,1\rangle \to -\frac{1}{\sqrt{53}}\langle 4,-6,1\rangle = \overrightarrow{QT}$$

T = ($2-\frac{4}{\sqrt{53}}$, $5+\frac{6}{\sqrt{53}}$, $3-\frac{1}{\sqrt{53}}$)

5. Consider the unit vectors $\vec{u}$ and $\vec{v}$:



a) Precisely describe in words the direction of $\vec{u} \times \vec{v}$. [2 pts]

$\vec{u}\times\vec{v}$ is a vector going into the paper, towards the floor.

b) Fill in the blank with either <, >, =, or "not enough info" [1]

$|\vec{u} \times \vec{v}|$ __>__ $\vec{u} \cdot \vec{v}$

negative

$\left\langle \frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}, 0\right\rangle$, $\langle 0,-1,0\rangle$

c) Draw the projection of $\vec{v}$ onto $\vec{u}$. [1]

(shown on $\vec{u}$)

$$\begin{vmatrix} i & j & k \\ 0.7 & 0.7 & 0 \\ 0 & -1 & 0 \end{vmatrix} = i\cdot 0 - j\cdot 0 + k\cdot \begin{vmatrix} 0.7 & 0.7 \\ 0 & -1 \end{vmatrix}$$

$-0.7 = -\frac{\sqrt{2}}{2}$