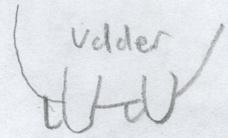
Analysis H – Deggeller / Gleason / Hahn Timothy Herthen is series-ly excited about this class

AtPS Quiz 2 2018/19 $\frac{30}{30}$ Period: 6

Calculators OK

1. The sum of a geometric series can be computed using the formula $S = \frac{a(1-r^n)}{1-r}$

   a) A first step in deriving the formula is to begin with a generic geometric series



$$S = a + ar + ar^2 + ar^3 + \cdots + ar^{n-1}$$

And multiply both sides by r:

$$Sr = ar + ar^2 + ar^3 + ar^4 + \cdots + ar^n$$

Using the two equations above, complete the derivation for the sum formula. Obviously, you already know the formula, so your answer will be graded on the clarity of your steps.

[cancel terms in difference]

$$Sr - S = ar^n + \cancel{ar^{n-1}} - \cancel{ar^{n-1}} + \cdots + \cancel{ar^2} - \cancel{ar^2} + \cancel{ar} - \cancel{ar} - a = ar^n - a$$

$$S(r-1) = a(r^n - 1) \quad \text{[factor out]}$$

$$S = \frac{a(r^n-1)}{r-1} = \boxed{\frac{a(1-r^n)}{1-r}}$$

   b) Find the sum of the following geometric series:

$$6 - 9 + \frac{27}{2} - \frac{81}{4} + \cdots + \frac{129140163}{32768}$$

$2^{-1}\ 2^0\ 2^1 \cdots 2^{15}$
$1\ 2\ 3 \cdots 17$

$$a = 6,\ r = -\frac{3}{2},\ n = 17$$

$$S = \frac{6\left(1-\left(-\frac{3}{2}\right)^{17}\right)}{1-\left(-\frac{3}{2}\right)} = \frac{6\cancel{+6}\left(\frac{3}{2}\right)^{17}}{5/2} = \frac{6 \cdot 2^{17} + 6 \cdot 3^{17}}{5 \cdot 2^{16}} = \boxed{\frac{3 \cdot 2^{18} + 2 \cdot 3^{18}}{5 \cdot 2^{16}}}$$

5k

   c) Use mathematical induction to prove that the geometric sum formula works for all positive n.

Prove: $a + ar + ar^2 + ar^3 + \cdots + ar^{n-1} = \frac{a(1-r^n)}{1-r}$

1) Base case where $n = 1$:

$$a = \frac{a(\cancel{1-r})}{\cancel{1-r}} = a$$

2) Assume true for $n = k$:

$$a + ar + ar^2 + \cdots + ar^{k-1} = \frac{a(1-r^k)}{1-r}$$

3) Prove for $n = k+1$:

$$a + ar + ar^2 + \cdots + ar^{k-1} + ar^k$$

$$= \frac{a(1-r^k)}{1-r} + ar^k = \frac{a(1-r^k) + ar^k(1-r)}{1-r} = \frac{a(1-r^k + r^k - r^{k+1})}{1-r}$$

$$= \boxed{\frac{a(1-r^{k+1})}{1-r}} \text{ as desired}$$

-0

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2. Find the sum of $1 - 2 + 3 - 4 + 5 - \cdots + 497 - 498 + 499 - 500$. Show your work!

$$\underbrace{\underbrace{1-2}_{-1} + \underbrace{3-4}_{-1} + 5 \cdots + \underbrace{497-498}_{-1} + \underbrace{499-500}_{-1}}_{500/2 = 250 \ -1\text{'s}}$$

$$S = 250(-1) = \boxed{-250}$$

or

$$1+3+5+\cdots+499 - (2+4+\cdots+500)$$

$$= 250^2 - 2\left(\frac{250(250+1)}{2}\right)$$

$$= 250^2 - 250(250+1)$$

$$= 250(250-250-1) = \boxed{-250}$$

3. Simplify completely (you can leave your answers in factored form, but your final expressions should not have factorials in them).

a) $\frac{(n^2-4)!}{(n-2)(n^2-5)!}$

$$= \frac{(n^2-4)}{(n-2)} = \frac{(n-2)(n+2)}{(n-2)}$$

$$= \boxed{n+2}$$

(assuming $n \neq 2$)

b) $\frac{(n!)^2}{(n-1)!(n+1)!} = \frac{n!\,n!}{(n-1)!\,(n+1)!}$

$$= \frac{n!}{(n-1)!} \cdot \frac{n!}{(n+1)!} = n \cdot \frac{1}{n+1} = \boxed{\frac{n}{n+1}}$$

4. The sum of the first 62 terms of a certain arithmetic series is 15,934. If the common difference of the sequence is 8, what is the first term of the series?

#1 #2 #3 #62

$$a + (a+8) + (a+8(2)) + \cdots + (a+8(61)) = 15934$$

$$62a + 8\left(\frac{61(61+1)}{2}\right) = 15934$$

15934

$$a = \frac{15934 - 8\left(\frac{61(61+1)}{2}\right)}{62} = \boxed{13}$$

5. Most mathematicians would say that the series below "diverges" and has no sum, because the sum goes to positive/negative infinity. However, if you HAD to give it a sum based on the patterns we've found in Pascal's Triangle, what would be the most appropriate value?

$1 - 5 + 15 - 35 + 70 - 126 + 210 - 330 + 495 - \ldots =$ $\frac{1}{32}$

row #

-5	1 -5 ...	$\longrightarrow 2^{-5}$
-4	1 -4 ...	
-3	1 -3 ...	
-2	1 -2 3 -4 ...	
-1	1 -1 1 -1 ...	$\longrightarrow 2^{-1}$? $2^{-5} = \frac{1}{32}$
0	1 0 0 0 ...	$\longrightarrow 2^0$
1	1 1 0 0 ...	$\longrightarrow 2^1$
	1 2 1 0 ...	
	1 3 3 1 ...	

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