Analysis Limits Quiz2

[30 pts]

27/30

Timothy says, "This quiz is as easy as 1, 1, 2, 3…"

1. Find the limit of the given sequence or say that it diverges. No need to justify. [2 ea]

a) $a_n = \dfrac{2\sin^2 n + 2\cos^2 n}{5}$

$\dfrac{2(1)}{5} = \boxed{\dfrac{2}{5}}$

b) $b_n = \left(5 + \dfrac{2}{n^2}\right)^4 \left(\dfrac{4n+3}{n}\right)$

$\boxed{5^4 \cdot 4}$

c) $b_n = \left\{\dfrac{3\cos(n)}{n}\right\}$

$\boxed{0}$

2. Are the following statements Always, Sometimes, or Never true? [1 ea]

a). If a sequence is always increaseing, then it diverges. sometimes

b) If a series $\sum a_n$ converges, then the sequence a_n converges to 0. always

c) An infinite series is convergent if the sequence of its partial sums is convergent. always

d) If a sequence a_n converges to 0, then the series $\sum a_n$ will converge. sometimes

3. Given $P_n = \dfrac{1}{3}, \dfrac{1}{15}, \dfrac{1}{35}, \ldots \dfrac{1}{4n^2 - 1}$

a) Determine whether P_n converges or diverges. Justify your answer using one of the tests we learned in class. [2]

$P_n = \dfrac{1}{4n^2-1} < \dfrac{1}{n}$ ← $n < 4n^2 - 1$ for $n \geq 1$

$\dfrac{1}{n} > \dfrac{1}{4n^2-1}$

$0 < \dfrac{1}{4n^2-1} < \dfrac{1}{n}$ so converges to 0 by comparison test

↑ converges to 0

b) Determine whether $\sum_{n=1}^{\infty} P_n$ converges or diverges. Justify your answer using one of the tests we learned in class. [2]

we have $P_n = \dfrac{1}{4n^2-1} < \dfrac{1}{n^2}$ for $n \geq 1$. Since

Since $P_n > 0$, $\sum P_n$ converges by the ~~p-series~~ test. (comparison test)

-1

4. [4] Use one of the tests from class to determine convergence/divergence for

$\{Hn\} = \left\{1 - \dfrac{1}{2} + \dfrac{1}{4} - \dfrac{1}{8} + \dfrac{1}{16} \ldots\right\}$ $= 2$? State the name of the test used, and justify its usage.

Alternating Series Test. If ~~the absolute value of an alternating series's~~ terms goes to 0, then the series converges.

$\lim\limits_{n\to\infty} \left|\dfrac{(-1)^{n+1}}{2^{n-1}}\right| = \lim\limits_{n\to\infty} 2^{-n+1} = 0$

-2

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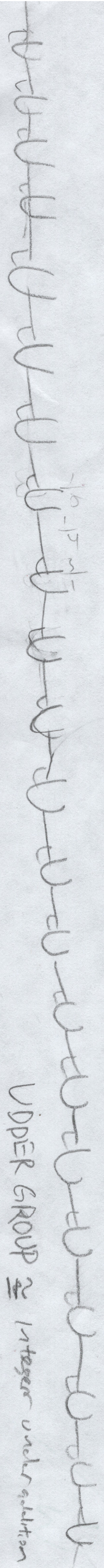
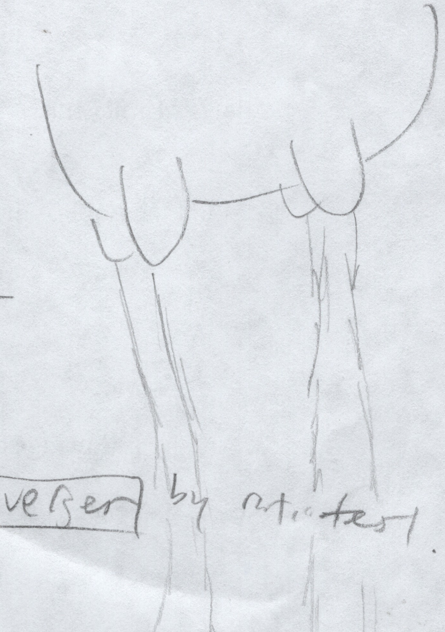
so $\sum\limits_{n=1}^{\infty} \dfrac{(-1)^{n+1}}{2^{n-1}}$ converges by AST

Alternatively, use the fact that $H_{2n-1} + H_{2n} = \dfrac{1}{2^{2n-1}}$, so $\sum H_n = \sum \dfrac{1}{2^{2n-1}} = \dfrac{2}{1-\frac{1}{4}}$ ~~$= \dfrac{2}{3}$~~ $\dfrac{2}{3}$ 3





5. [4] Use the Ratio test to determine convergence of $\sum_{n=0}^{\infty} \frac{n!}{5^n}$

we have $\frac{p_{n+1}}{p_n} = \frac{\frac{(n+1)!}{5^{n+1}}}{\frac{(n)!}{5^n}} = \frac{n+1}{5}$

Since $\lim_{n\to\infty} \frac{n+1}{5} > 1$, the series $\boxed{\text{diverges}}$ by ratio test.

6. Consider the sequence $t_n = \frac{-n}{2+3n}$, starting with n=1.

a) [1] The limit of this sequence (as n goes to infinity) is $-\frac{1}{3}$

b) [2] Is the sequence always increasing, always decreasing, or neither? Justify your answer using algebra.

$t_{n+1} = \frac{-(n+1)}{2+3(n+1)} = \frac{-n-1}{5+3n}$ $\quad \frac{-n}{2+3n} \cdot \frac{5+3n}{5+3n} = \frac{-5n-3n^2}{(5+3n)(2+3n)}$

Compare numerator $-3n^2-2-5n < -5n-3n^2$

$-2 < 0$

$\frac{-n-1}{5+3n} \cdot \frac{2+3n}{2+3n} = \frac{-2n-2-3n^2-3n}{(5+3n)(2+3n)}$ $\quad \frac{-3n^2-2-5n}{(5+3n)(2+3n)} \overset{?}{<} \frac{-5n-3n^2}{(5+3n)(2+3n)}$

positive

$\boxed{\text{always decreasing}}$

c) [2] Using your answer from b (and some more logic) prove that the sequence converges. Write a conclusion statement showing why your proof is valid.

we have $n < n+\frac{2}{3} \to \frac{n}{n+\frac{2}{3}} < 1 \to \frac{n}{3n+2} < \frac{1}{3} \to \frac{-n}{3n+2} > -\frac{1}{3}$

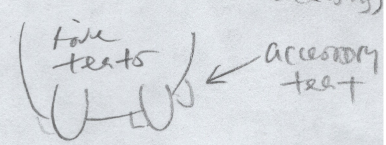
so t_n is bounded below by $-\frac{1}{3}$, and given (b), must therefore converge

d) [3] Prove that your answer to part "a" is right by showing that after a certain term (M), the sequence will be within a neighborhood of radius $\varepsilon = 1/30$ from your limit. Write a sentence at the end summarizing your logic.

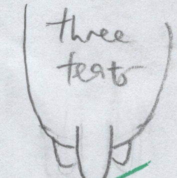
we need

$\frac{-n}{2+3n} < -\frac{1}{3} + \frac{1}{30}$ (other side unnecessary because always decreasing)

$\frac{-n}{2+3n} < -\frac{9}{30} = -\frac{3}{10}$



$\frac{-n}{2+3n} = -\frac{3}{10} \to -10n = -3(2+3n) \to -10n = -6-9n \to 6 = n$ so



if $M = 6$, for all $n > M$ we have $t_n < -\frac{1}{3} + \epsilon$,
so t_n is within ϵ of $L = -\frac{1}{3}$.

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