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Note to Self = look up ISP & ZFC lol

1. Here is a list of infinite sets, coupled with an operation. For each, say "yes" or "no" to whether or not it is a group.

a) integers under addition

yes

b) odd integers under addition

no

c) real numbers under multiplication

no

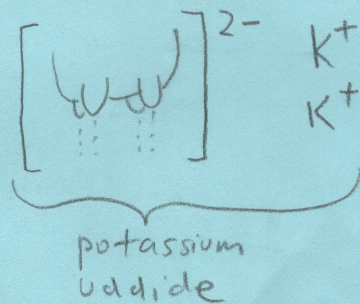
d) integer powers of 5 under multiplication

yes

e) rotation by an integer number of degrees

yes

f) complex numbers under addition

yes

2. Which two groups from #1 (refer to them by letter) are isomorphic to one another? a and d.

3. Give a set of elements that forms a group under multiplication, that is isomorphic to the rotation group of a pentagon.

$$S = \{ 1, \text{cis } \frac{2\pi}{5}, \text{cis } \frac{4\pi}{5}, \text{cis } \frac{6\pi}{5}, \text{cis } \frac{8\pi}{5} \}$$

4. The complex number $z = \frac{(-4+4i)^5}{(\sqrt{3}-i)^{12}}$ can be simplified to $a + bi$ form where a and b are integers. Find a and b .

$$-4+4i = 4\sqrt{2} \cdot \text{cis } \frac{3\pi}{4}$$

$$\sqrt{3}-i = 2 \cdot \text{cis } \left(-\frac{\pi}{6}\right)$$

$$z = \frac{(4\sqrt{2})^5 \cdot \text{cis } \frac{15\pi}{4}}{(2)^{12} \cdot \text{cis } \left(-\frac{12\pi}{6}\right)} =$$

$$(4\sqrt{2})^5 = (2^{5/2})^5 = 2^{25/2}$$

$$\frac{\sqrt{2} \cdot 2^{25/2} \cdot \left(\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}i\right)}{2^{12} \cdot 1} = 1-i$$

$$a = \underline{1}$$

$$b = \underline{-1}$$

Logang

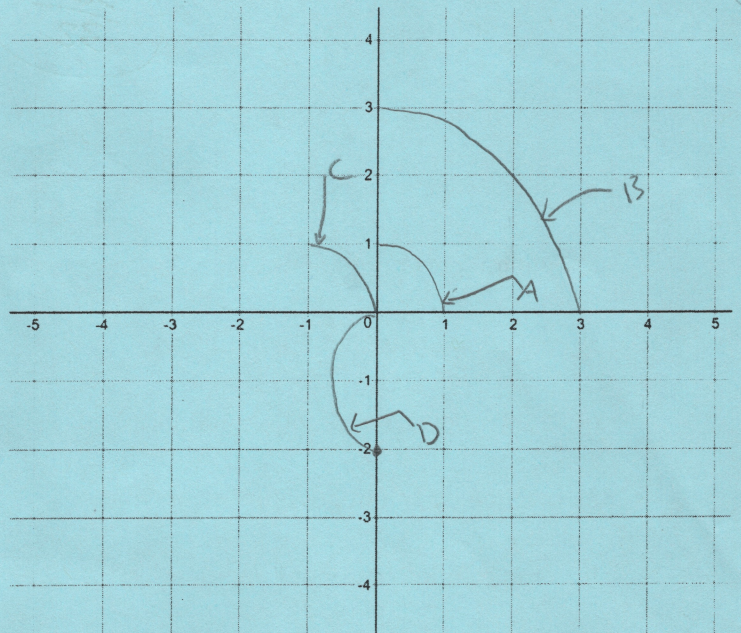
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5. On the coordinate axis on the right...

- a) Plot the set of all complex numbers z in the first quadrant where $|z|=1$. Label this set as "A".

Use these numbers to obtain your answers for parts (b)-(d) below.

- b) Plot the set of all complex numbers $3z$. Label this set as "B".
- c) Plot the set of all complex numbers $(z-1)$. Label this set as "C".
- d) Plot the set of all complex numbers $(z-1)^2$. Label this set as "D".



$$(e^{i\theta} - 1)^2 = e^{2i\theta} - 2e^{i\theta} + 1$$

$$= \cos 2\theta + i \sin 2\theta - 2\cos \theta - 2i \sin \theta + 1$$

$$(-1+i)^2 = 1 - 2i + i^2 = 1 - 2i - 1 = -2i$$

$$r = -2\cos \theta$$

Lemniscate

$$\rightarrow (\cos 2t - 2\cos t + 1, \sin 2t - 2\sin t) \text{ from } 0 \leq t \leq \frac{\pi}{2}$$

6. The matrix multiplication below shows a transformation matrix's effect on quadrilateral ABCD.

$$\begin{bmatrix} 2 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 2 & 2 \\ 0 & -1 & 0 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 2 & 4 & 4 \\ 0 & 1 & 0 & -2 \end{bmatrix}$$

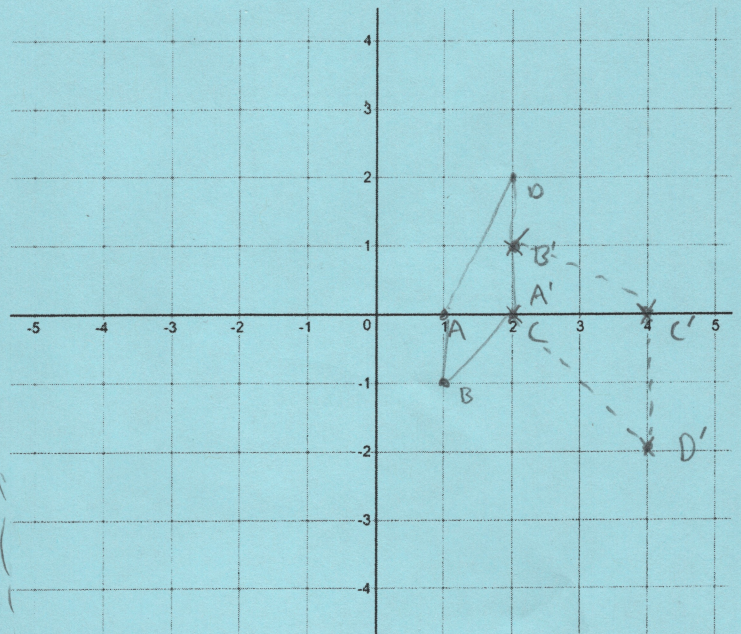
Fig 2:

Three-legged spider

- a) On the grid to the right, graph the quadrilateral ABCD and label its vertices A, B, C, and D.

- b) Perform the indicated transformation and label the resulting image, with vertices A', B', C', and D'.

- c) Describe the effect of the transformation matrix (use specific math vocabulary).



$$(\cos \theta + i \sin \theta - 1)^2$$

$$= \cos^2 \theta - \sin^2 \theta + 1 + 2i \sin \theta \cos \theta - 2i \sin \theta - 2 \cos \theta$$

$$\cos 2t - 2\cos t + 1$$

$$\left(\frac{\sqrt{2}}{2} - 1, \frac{\sqrt{2}}{2}i\right)$$

$$\left(\frac{\sqrt{2}}{2} - 1 + \frac{\sqrt{2}}{2}i\right)^2 = \left(\frac{\sqrt{2}}{2} - 1\right)^2 +$$

-D