eggstf

Analysis Quiz 2018-19
Matrix Quiz 3 [30]

$\frac{30}{30}$

Herreshoff vs. Timothy H[illegible]

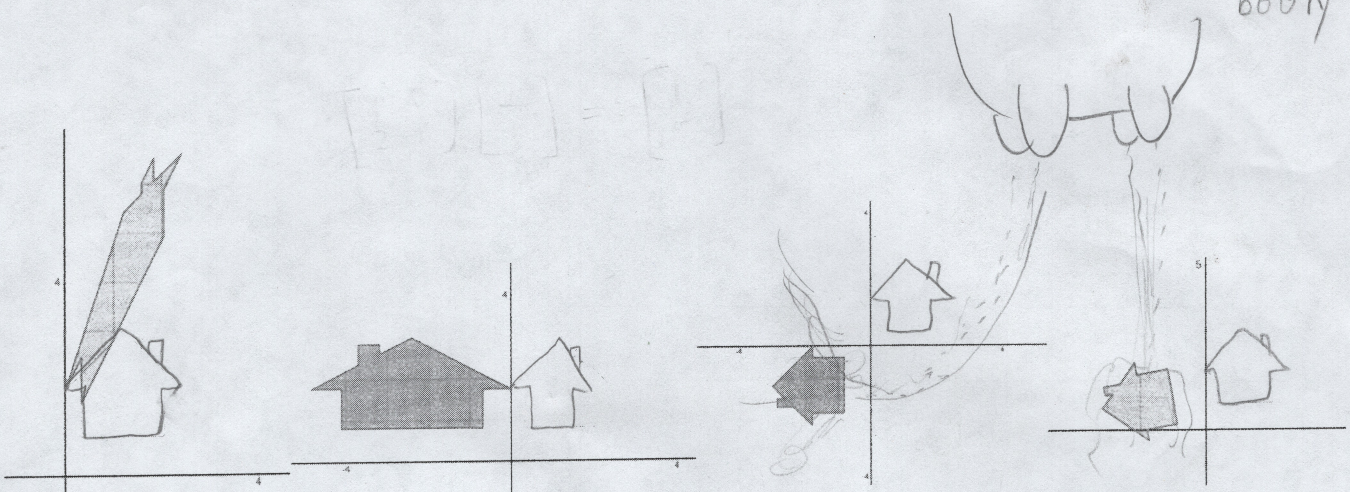
tf ff

botfly

botfly

1. Write a matrix that would turn the original house in the 1st quadrant into its new image. In some cases you might have to make some approximations which is fine. As long as you're close you'll get full credit. [3 each]

botfly



a) b) `c) d)

a) $\begin{bmatrix} 1 & 0 \\ \cancel{2} & 1 \end{bmatrix}$ $\frac{1}{2}$ b) $\begin{bmatrix} -2 & 0 \\ 0 & 1 \end{bmatrix}$ c) $\begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$ d) $\begin{bmatrix} \cos 100^\circ & -\sin 100^\circ \\ \sin 100^\circ & \cos 100^\circ \end{bmatrix}$

-1 2 botfly rotifer grotesque

e. Write a matrix that would map every point on the house to a point on the line that passes through the origin and (4, -3) [3]

$$\boxed{M = \begin{bmatrix} 4\pi & 4\pi \\ -3\pi & -3\pi \end{bmatrix}}$$

$$\begin{bmatrix} 4 & 4 \\ -3 & -3 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4x+4y \\ -3x-3y \end{bmatrix} = \begin{bmatrix} 4(x+y) \\ -3(x+y) \end{bmatrix}$$

SUMMON THE ROTIFERS

2. Perform the operation (2 + 5i)(3 – 2i) using matrices. Show how your answer can be converted back into a + bi form. [3]

$$2+5i \Leftrightarrow \begin{bmatrix} 2 & -5 \\ 5 & 2 \end{bmatrix} ; \quad 3-2i \Leftrightarrow \begin{bmatrix} 3 & 2 \\ -2 & 3 \end{bmatrix}$$

$$\begin{bmatrix} 2 & -5 \\ 5 & 2 \end{bmatrix}\begin{bmatrix} 3 & 2 \\ -2 & 3 \end{bmatrix} = \begin{bmatrix} 16 & -11 \\ 11 & 16 \end{bmatrix} \Leftrightarrow \boxed{16+11i}$$

-0

4. Below are generators for two different groups. For each, state i) the transformation(s) represented by the matrix (or matrices); ii) the order (size) of the group and iii) a different group that it is isomorphic to. [12 total] [4 each]

a) $\begin{bmatrix} \cos 20^\circ & -\sin 20^\circ \\ \sin 20^\circ & \cos 20^\circ \end{bmatrix}$ i) rotation ccw by 20° ii) 18

$\frac{360^\circ}{20^\circ} = 18$

iii) C_{18} (cyclic group of order 18)

b) $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ and $\begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}$

i) shear in x by factor 1 and sheer in x by factor −1 ii) $\aleph_0$ (countable infinity)

iii) $\{\mathbb{Z}, +\}$ (integers under addition)

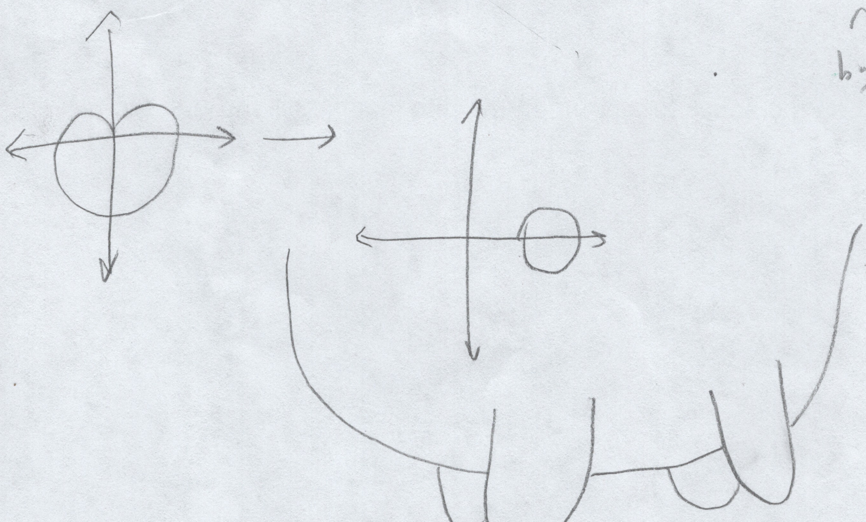
$\begin{bmatrix} 1 & x \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & y \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & x+y \\ 0 & 1 \end{bmatrix}$ $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$ $\begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix}$

5. Express the following as a composition of 2 common matrix transformations. (make sure your order is correct) [4]

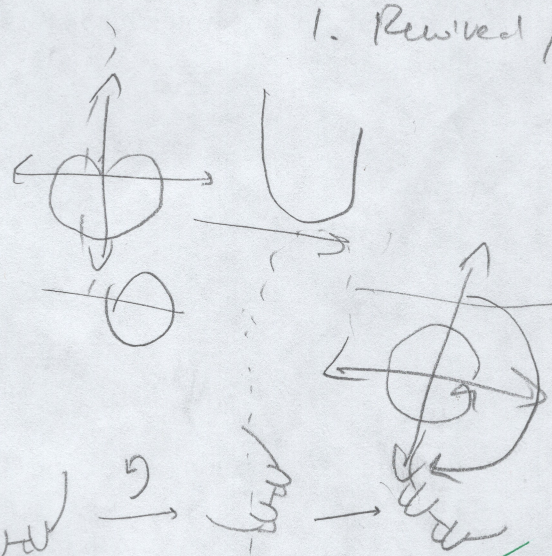
$M = \begin{bmatrix} -\cos 50^\circ & -\sin 50^\circ \\ -\sin 50^\circ & \cos 50^\circ \end{bmatrix}$

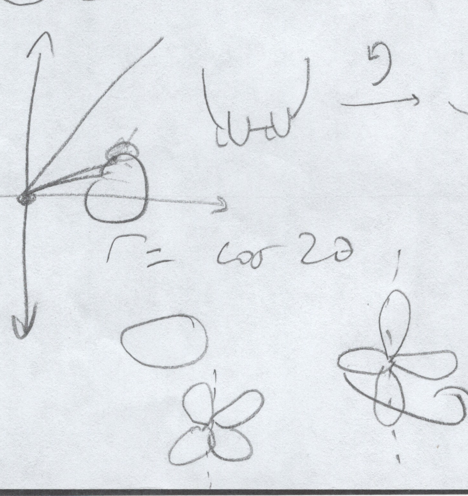
$M = \underbrace{\begin{bmatrix} \cos 50^\circ & -\sin 50^\circ \\ \sin 50^\circ & \cos 50^\circ \end{bmatrix}}_{\text{rotation ccw by } 50^\circ} \underbrace{\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}}_{\text{reflection over y axis}}$

1. Revised p1/2



$\cos(\theta) + \sqrt{1 - 0.5^2 \sin^2(\theta)}$





$r = f(\theta, \phi)$

$r = \cos 2\theta$

$f(\theta, \phi) = \theta$

−0