Analysis H – Deggeller / Gleason / Hahn
Delta-Epsilon Limits Quiz
NO CALCULATORS

15/15

Timothy Herten has no limits
Period: 6

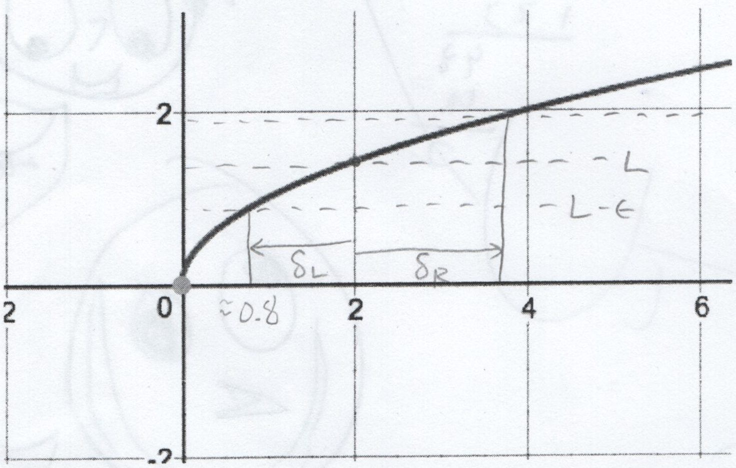
1. Fill in the blanks: [3 pts]

$\lim_{x \to c} f(x) = L$ if and only if for every $\epsilon > 0$, no matter how small, there exists a(n) $\delta > 0$ such that if x is within δ units of c, then $f(x)$ is within ϵ units of L.

2. Refer to the graph of f(x) below. Estimate all answers to the nearest tenth.

a. $\lim_{x \to 2} f(x) =$ 1.4 [1 pt]

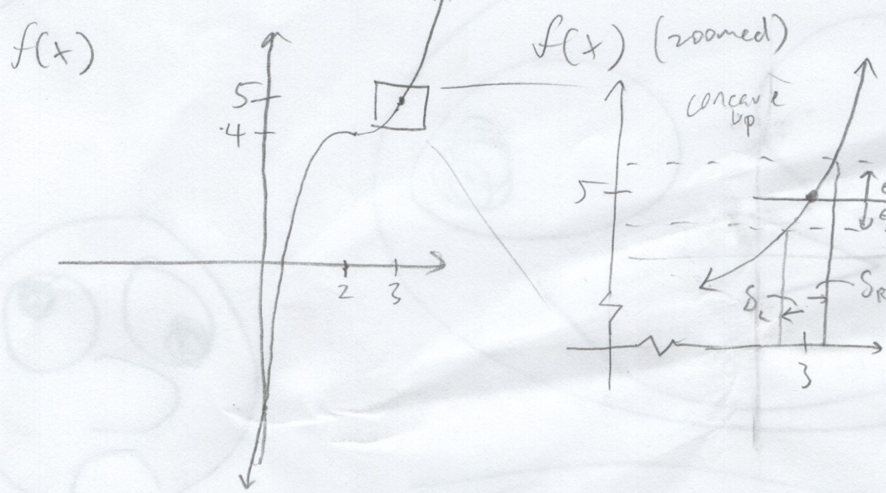
b. Given $\varepsilon = 0.5$, use the graph to *estimate* the largest possible value of δ that will satisfy the limit definition. Draw on the graph to clearly indicate important lines and measurements that you considered. [4 pts]



$\min\{\delta_L, \delta_R\} = \delta_L$

so $\delta_{max} = \delta_L \approx 2 - 0.8 = \boxed{1.2}$

3. Given the function, $f(x) = (x-2)^3 + 4$, complete a delta epsilon proof to prove: $\lim_{x \to 3} f(x) = 5$. [7 pts]



The picture shows $\delta_R < \delta_L$, so we want to find δ_R in terms of ϵ and set δ to that.

$(x-2)^3 + 4 = 5 + \epsilon \rightarrow (x-2)^3 = 1 + \epsilon \rightarrow x - 2 = \sqrt[3]{1+\epsilon} \rightarrow x = 2 + \sqrt[3]{1+\epsilon}$.

Then $\delta_R = x - 3 = 2 + \sqrt[3]{1+\epsilon} - 3 = \sqrt[3]{1+\epsilon} - 1$. Thus,

For any $\epsilon > 0$, let $\delta = \sqrt[3]{1+\epsilon} - 1$. Then $\delta > 0$ and $0 < |x-3| < \epsilon \rightarrow |f(x) - 5| < \epsilon$

Therefore, $\lim_{x \to 3} f(x) = 5$

Q.E.D.

$f(x) = x^3 - 5x + 2$

$-2 \,|\, 1 \quad -5 \quad 2$
$\quad\; -2$
$\quad 1 \quad -7$

$\lim_{x \to -2} \dfrac{x^3 - 5x + 2 - 4}{-x+2} = \lim_{x \to -2} \dfrac{(x+2)(x^2 - 2x - 1)}{-x+2}$

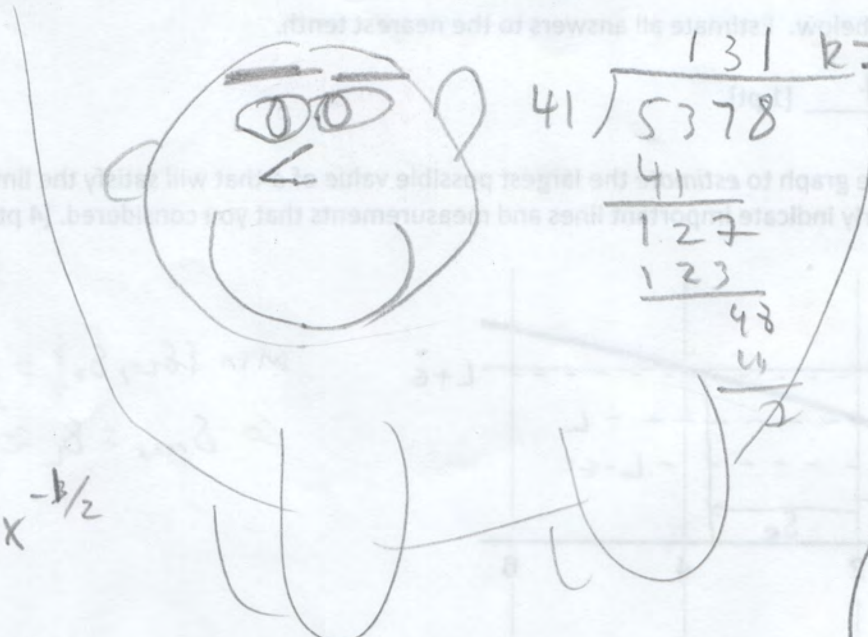
$= \lim_{x \to -2} x^2 - 2x - 1 = \boxed{7}$

$-2 \,|\, 1 \quad -5 \quad 2$
$\quad\quad -2 \quad 14$
$\quad 1 \quad -7 \quad 16$

$x - 7$

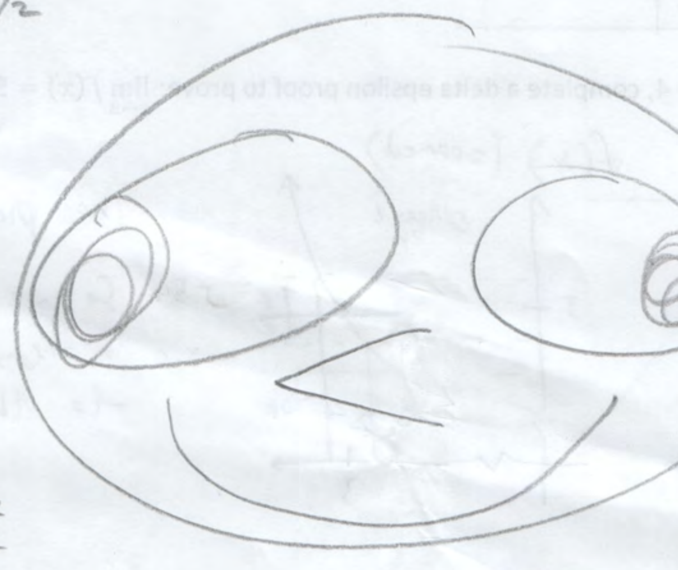
$-2 \,|\, 1 \quad 0 \quad -5 \quad -2$
$\quad\quad -2 \quad 4 \quad +2$
$\quad 1 \quad -2 \quad -1 \quad 0$

$41 \overline{)5378}$ = 131 R27
41
127
123
48
41
7



$\frac{3}{2} x^{-1/2}$

$\frac{3}{2} \left(\frac{1}{4}\right)^{-1/2}$



$-2 \,|\, 1 \quad 2 \quad 0 \quad -3 \quad 2$
$\quad\quad\quad -2 \quad 4 \quad 2$
$\quad 1 \quad -2 \quad 1$

$\frac{9}{8} \quad -\frac{9}{2}$