I've got all my life to live, I've got all my love to give, I will derive! Brandon C

Period E

Analysis Calculus Quiz 18/19 – **No Calculators!**

$\frac{50}{50}$

1. Iron Hans decided to keep track of his velocity as he biked the second leg of his triathlon. He later realized it could be modeled by the function $f(x) = \frac{-2^x}{3} + 18$ where $f(x)$ is measured in miles per hour and x is measured in hours.

a) What was the average rate of change of his velocity over the time interval x: [0, 4] ? Include units

$\frac{1-2^4}{3 \cdot 4} = -\frac{5}{4} \frac{mi}{h^2}$

b) Hans used his calculator to approximate $f'(2) = -0.924$. Use words to explain what this number means in the context of the problem. Include units in your explanation.

at 2 hours, Hans's velocity was decreasing by 0.924mph per hour

c) Why can we be sure that the Intermediate value theorem applies to f(x) over x: [0, 4] ?

f(x) is continuous, no undefined or indeterminate values, and $f(0) \neq f(4)$

d) State one thing that the Intermediate Value Theorem would guarantee for this situation.

Knowing Hans's velocity at two points in time, we know he must have been at any velocity between them between the points in time ~~ther~~ OK

every?

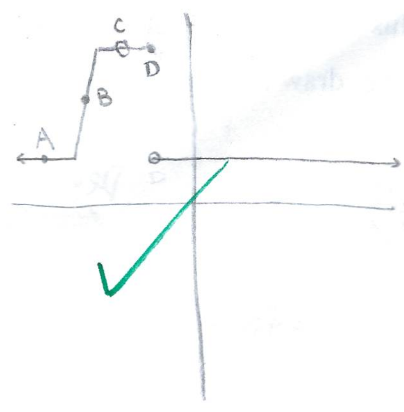
2. In the space on the right, sketch a graph of a function with x values A, B, C, and D (in that order) that satisfies the following conditions. Label the points of course!!

i) The derivative at x=A is zero.

ii) The derivative at x=B is a very large and positive number.

iii) The function has a limit at x=C but no output.

iv) The function has an output but no limit at x=D.



3. The sewer pipes in my front yard exploded last week! The utility company started working on the pipe at 1:00, but had workers coming in and out over the next few hours (I know because I kept track). Below are the number of workers on the project at 20 minute intervals after 1:00.

minutes after 1:00 (t)	0	20	40	60	80
Workers (y)	1	4	6	5	2

a) Approximate the derivative at t=60 minutes. Include units.

$\frac{2-6}{80-40} = -\frac{1}{10}\ \frac{\text{workers}}{\text{min}}$

b) Use the trap rule to estimate the definite integral over the 80-minute period as accurately as possible and state units.

$20\left(\frac{1}{2}+4+6+5+\frac{2}{2}\right) = 20(16.5) = 330$ workers·min

4. Evaluate the following limits or state that they do not exist. Show algebraic work for full credit.

$$a)\lim_{x\to 3}\frac{\frac{1}{x^2}-\frac{1}{9}}{x-3}$$

$\rightarrow \frac{\frac{9}{9x^2}-\frac{x^2}{9x^2}}{x-3}$

$\lim_{x\to3} -\frac{(x+3)(x-3)}{9x^2(x-3)}$

$-\frac{x+3}{9x^2}$ $\quad -\frac{6}{81} \quad \boxed{-\frac{2}{27}}$

$$b)\lim_{x\to 3}\frac{\frac{1}{x^2}+\frac{1}{9}}{x-3}$$

$\frac{\frac{1}{9}+\frac{1}{9}}{3-3} = \frac{\frac{2}{9}}{0} \neq \frac{0}{0}$

as $x\to3^-$, $f(x)\to-\infty$

$2.999\ldots \rightarrow \frac{\frac{1}{8.999..}+\frac{1}{9}}{-0.000\ldots1} = -\infty$

$3.000\ldots1 \rightarrow \frac{\frac{1}{9.000..1}+\frac{1}{9}}{0.000\ldots1} = \infty$

as $x\to3^+$, $f(x)\to\infty$

$\boxed{\text{no limit}}$

5. The following equation implies a certain function and important information about that function. Use your calculus knowledge to interpret the equation's hidden information. To demonstrate your understanding, draw an accurate picture of a function, a tangent line to the function, and a specific point of tangency.

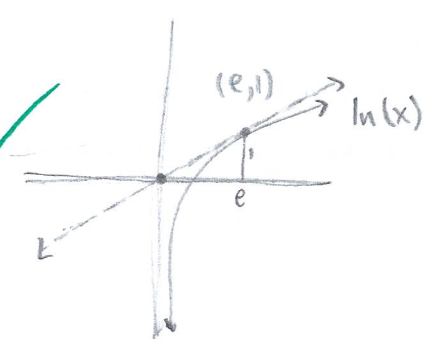
$$\lim_{x\to e}\frac{\ln x-1}{x-e}=\frac{1}{e}$$

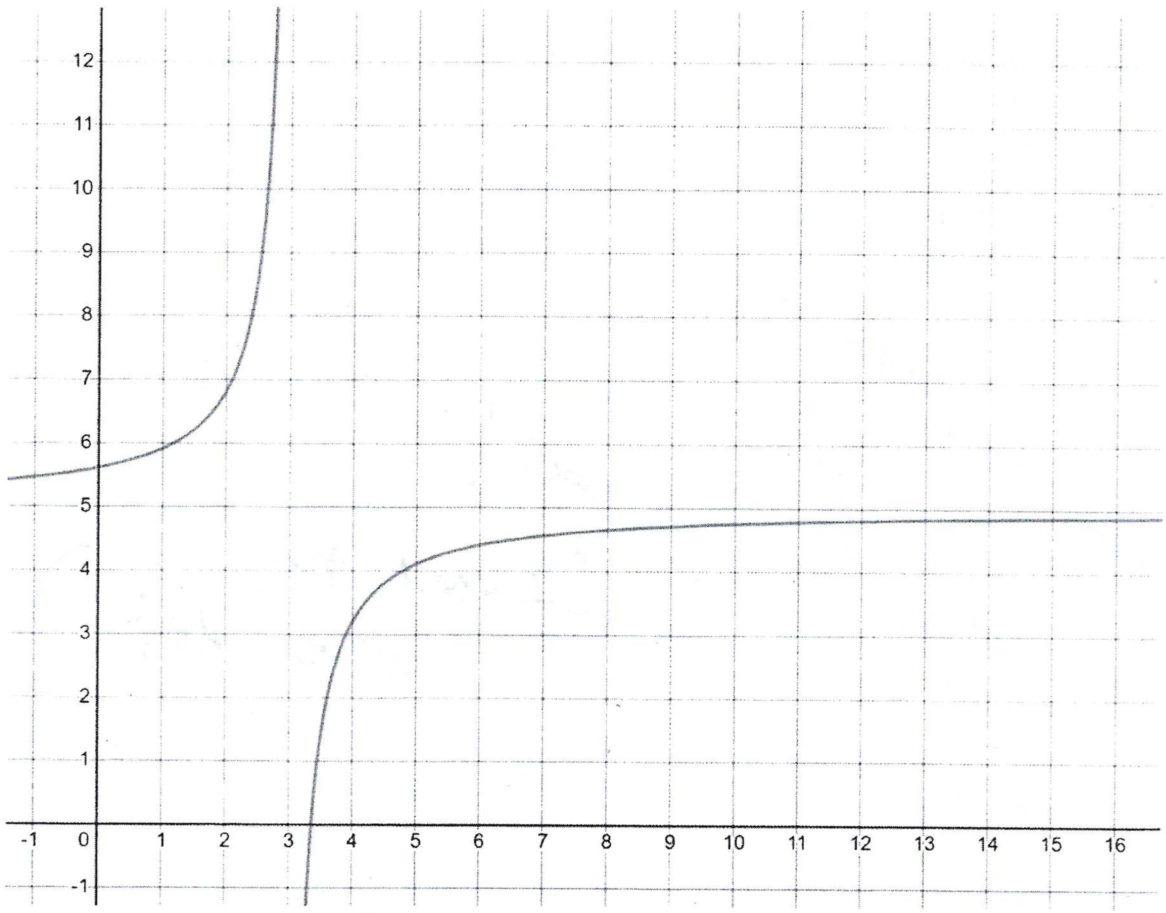
Derivitve of lnx at e is $\frac{1}{e}$

$\cancel{\ln(e)=1}$

$c=e$

$f(x)=\ln(x)$ $\quad \lim_{x\to c}\frac{f(x)-f(c)}{x-c} = \boxed{f'(e)=\frac{1}{e}}$





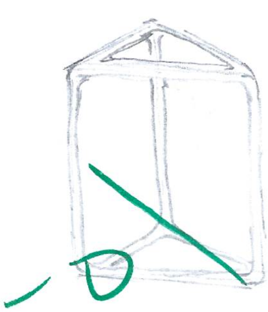
6. The graph above shows f(x).

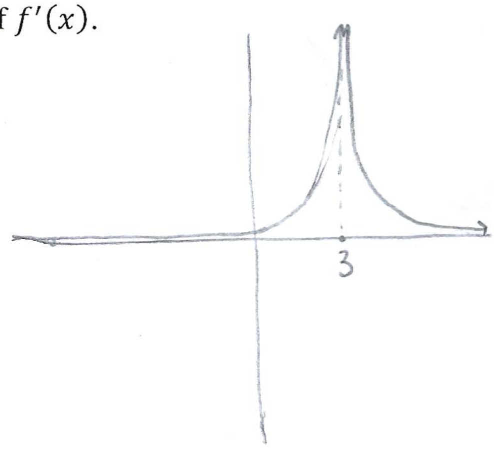
a) Using the graph as reference, fill in the blanks (with numerical values, not variables). Show your work on the graph so that I can follow your reasoning.

The limit of f(x) as x approaches infinity seems to be __5__ because if x > __7__ then f(x) will be within 0.5 units of its limit.

The limit of f(x) as x approaches __3__ from the left side seems to be positive infinity because if x is within __0.25__ units of __3__ from the left side then f(x) > 10

b) In the space below, sketch a graph of $f'(x)$.





7. Use the formal definition of the derivative of a function to prove:

$$\frac{d}{dx}\sqrt{x} = \frac{1}{2\sqrt{x}}$$

(hint: conjugate!)

$$\lim_{x\to c} \frac{\sqrt{x}-\sqrt{c}}{x-c}$$

$$\lim_{x\to c} \frac{(\sqrt{x}-\sqrt{c})(\sqrt{x}+\sqrt{c})}{(x-c)(\sqrt{x}+\sqrt{c})}$$

$$\lim_{x\to c} \frac{x-c}{(x-c)(\sqrt{x}+\sqrt{c})}$$

cool!

$$\frac{1}{\sqrt{x}+\sqrt{x}}$$

(I used the other def)

$$\frac{1}{2\sqrt{x}}$$

8. Given $f(x) = 2x^3 - 4x + 5$

a) Use the power rule to find $f'(x)$.

$6x^2 - 4$

✓

b) Find all points on f(x) where the instantaneous rate of change is 21.

$6x^2 - 4 = 21$

$x^2 = \frac{25}{6}$

$x = \pm\frac{5}{\sqrt{6}} = \pm\frac{5\sqrt{6}}{6}$

✓

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