Analysis H – Deggeller / Gleason / Hahn
Unit 5: Growth Big Quiz
Non-Calculator Section

Powerful Student: Timothy Herben
Period: ~~4~~ 6

50/50

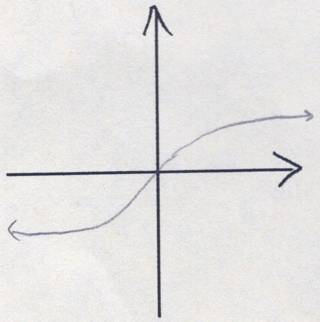
1. Consider the power function $f(x) = kx^{\frac{a}{b}}$, where k, a and b are non-zero integer constants, and a and b do not have mutual factors (other than 1, duh). Also, $a \neq b$. For each problem, give values of k, a and b that will make the statement true, and then sketch the corresponding power function. If it is not possible, write "not possible" and leave the graph blank.

a) The function is in quadrants 1 and 3, and crosses through the origin.

k = 1

a = 1

b = 3



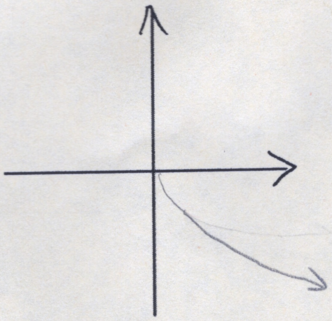
$f(x) = \sqrt[3]{x}$

b) The function is only in quadrant 4, and $f(2) > f(5)$.

k = −1

a = 3

b = 4



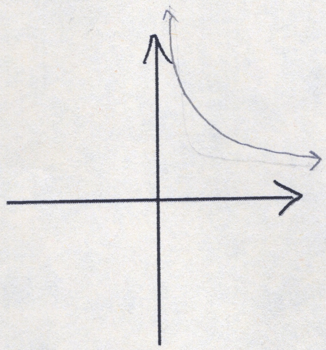
$f(x) = -x^{\frac{3}{4}}$

c) The function is only in quadrant 1, and has an asymptote

k = 1

a = −3

b = 2



$y = x^{-3/2}$

$= \frac{1}{x^{3/2}}$

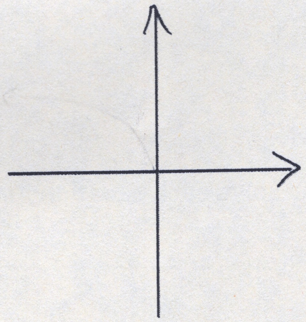
d) The function is only in quadrant 2 and hits (0, 0).

Not Possible!

k = ______

a = ______

b = ______



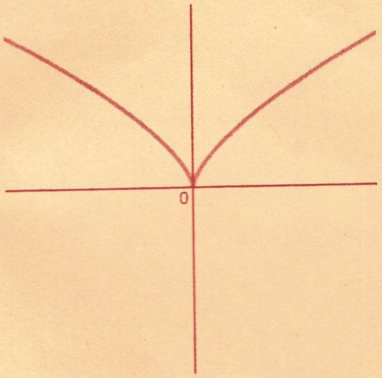
2. Consider the logistic function $y = \frac{A}{1+Be^{-x+C}}$. Find the point of inflection in terms of A, B, and/or C.

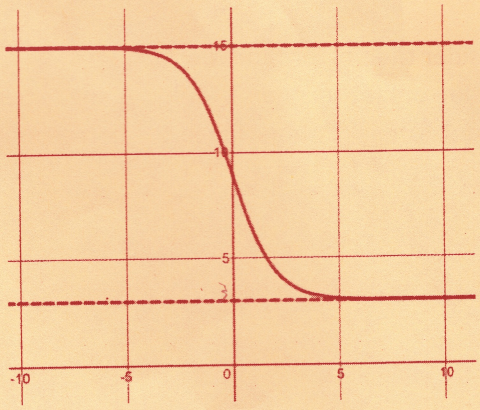
$y = \frac{A}{1+e^{-x+C+\ln B}} = \frac{A}{1+e^{-(x-(-\ln B)}}$

$\boxed{(C+\ln B, A/2)}$

/ −0

3. Write an equation that will match each graph below. You may not use piecewise functions or absolute value sy in your answers.





Equation: $y = x^{\frac{4}{5}}$

Equation: $\frac{12}{1+e^x} + 3$

4. Solve for n: $log_2 3 \cdot \log_3 4 \cdot log_4 5 \ldots log_n(n+1) = 10$

$\log_a b = \frac{\log b}{\log a}$ $\qquad \log_3 9 = \frac{\log 9}{\log 3} = \frac{2\log 3}{\log 3} = 2$

$\log_2 3 \cdot \log_3 4 \cdot \ldots \cdot \log_n(n+1) = \frac{\cancel{\log 3}}{\log 2} \cdot \frac{\cancel{\log 4}}{\cancel{\log 3}} \cdot \ldots \cdot \frac{\log(n+1)}{\cancel{\log n}}$

$= \frac{\log(n+1)}{\log 2} \to \frac{\log(n+1)}{\log(2)} = 10, \quad \log_2(n+1) = 10, \quad \boxed{n = 2^{10} - 1 = 1023}$

$\log_9 4 + 1 = \log_3 6 \to \log_3 2 + \log_3 3 = \log_3 6$

5. Solve each equation for x.

a) $ln[log(2 + log_2(x+1))] = 0$

$\log(2 + \log_2(x+1)) = 1$

$2 + \log_2(x+1) = 10$

$\log_2(x+1) = 8$

$x + 1 = 2^8$

$\boxed{x = 2^8 - 1 = 255}$

b) $\log_9(x-1) + \log_9(2x-1) = \log_3(x+1)$

$\log_9((x-1)(2x-1)) = \log_3(x+1)$

$= \log_9((x+1)^2)$

$(x-1)(2x-1) = (x+1)^2$

$2x^2 - 2x - x + \cancel{1} = x^2 + 2x + \cancel{1}$

$x^2 - 5x = 0$

$\boxed{x = 5}$ (0 is extraneous)

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Interested Student: Timothy Herhen Per: 6

Potentially useful formulas:

$$FV = R\frac{(1+i)^n - 1}{i} \qquad PV = R\frac{1-(1+i)^{-n}}{i}$$

6. If you invest $3000 (one-time investment) into an account that earns 6% interest, compounded monthly, how long will it take for the account balance to equal $7000?

$$3000(1.005)^n = 7000$$

$$\lceil n \rceil = \lceil \log_{1.005} \tfrac{7}{3} \rceil = \lceil 169.88 \rceil = \boxed{170 \text{ months}}$$

7. Diane is a spry, 22-year old teacher at the dawn of her career. She calculates that if she lives on a good budget, she can invest $400/month into the stock market. If she continues her diligent saving and investing over her career, how much will she have in her stock market account when she retires at age 65? From Investopedia: The average annual return for the S&P500 (a major stock market indicator) for the time period 1928-2016 is approximately 10%. So let's just use 10% for Diane's calculations as well.

annual return = 10%, monthly return = $\frac{10\%}{12}$, months = $(65-22)\cdot 12 = 516$

$$FV = 400\frac{\left(1+\frac{10\%}{12}\right)^{516} - 1}{\frac{10\%}{12}} = \boxed{\$3427116.43}$$

8. In A.D. 225, a Roman judge produced the first known mortality table for "annua," which were lifetime stipends made once per year in exchange for a lump-sum payment. Maximus, a recent retiree, is 65 years old, and wants to stabilize his cash flow. In exchange for 600 silver coins paid today, the Bank of Romerica is offering him an annual payment of 22 silver coins per year over the next 30 years. What annual interest rate is Maximus receiving in this deal?

$$PV = 22\cdot\frac{1-(1+i)^{-30}}{i} = 600$$

$$\frac{1-(1+i)^{-30}}{i} = \frac{600}{22} \longrightarrow \boxed{i \simeq 0.626\%}$$

/ -0

9. Diane (from problem 7) has a coworker named Donald. Like anyone, Donald has strengths and weaknesses, but one of his big weaknesses is fancy cars. Donald also calculates that he has an extra $400 per month, and decides to put it towards a new Tesla Model 3, which will cost him $46,000, which he doesn't have. If he borrows $46,000 at a 4.74% annual rate and commits to making $400 payments each month, how many months will it take for Donald's loan to be paid off?

monthly rate $= \frac{4.74\%}{12} = 0.00395$

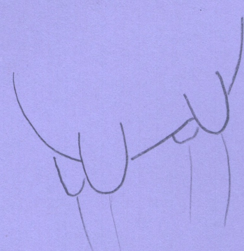
$$46000 = 400 \cdot \frac{1-(1.00395)^{-n}}{0.00395}$$

$$-n = \log_{1.00395}\left(1 - \frac{460}{4} \cdot 0.00395\right)$$

$$= -153.617 \rightarrow \boxed{154 \text{ months}}$$

10. Looking back at problem 9, how much profit did the bank make from Donald's loan? (In today's dollars. Don't worry about PV or FV.)

$154 \cdot \$400 - \$46000 = \boxed{\$15600}$



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