Analysis Midterm 1 19-20 _____ is independent of grade
Deggeller/Gleason/Tantod
ATPS [36 pts]

1. Express all of the following as a compact expression of as few terms as possible [3 pts each]:

$a)$ $F_{20} + F_{21} + F_{22} + \cdots + F_{90} + F_{91}$ _____

$b)$ $\binom{77}{1} + \binom{77}{3} + \binom{77}{5} + \cdots \binom{77}{75} + \binom{77}{77}$ _____

$c)$ $\sum_{k=5}^{\infty} \frac{3}{7^k}$ _____

$d)$ $\frac{\binom{n}{n-2}}{\binom{n-2}{n-4}}$ (assume n is a whole number bigger than 4). _____

$e)$ $\prod_{n=1}^{k} (2n)$ _____

2. The sum of all the multiples of 2 from 2 to 1,700 = 723,350

The sum of all the multiples of 17 from 17 to 1,700 = 85,850

a) In one sentence, explain why the sum of all the multiples of 2 and 17 between 2 and 1,700 is not 723,350 + 85,850. [2 pt]

b) Find the sum of all the multiples of 2 and 17 between 2 and 1,700 (no need to simplify) [3 pts]

3. Pierre thinks he has a formula for the sum of the first n cubes: $\frac{(n(n+1))^2}{4}$ where n is a whole number.

   a) Show at least two non-zero examples of specific positive whole numbers n where Pierre is right. [2]

   b) Now that you've established a couple base cases, use induction to show that Pierre is indeed correct. Please be proper about your proof so your steps are obvious. [6]

4. If you were to expand the trinomial $(2a - 3b + c)^{100}$ you'd have a lot of terms. One of them would have an $a^{10}b^{70}c^{20}$ in it. What would the coefficient of this term be? [4]

5. Consider the "even" triangle below (first 4 rows given).

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              2
           4     6
        8     10    12
     14   16    18    20
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Find the last term of the 53rd row. Show the work that leads to your answer. [4]

Probability Section: [36 points]

1. A fair 6-sided die is being rolled twice. What is the probability that the same face (#) comes up twice in a row? [2]

2. I write the eleven letters in "Mississippi" in random order. What is the probability that the last letter is an i? [2]

3. Erica wrote the five letters of her name in random order. What is the probability that they are all in the right place? [2]

4. I pick six cards at random from a standard deck of 52 cards.
   a) What is the probability that there are exactly three hearts and two clubs in my hand? [3]

   b) What is the probability I have three of a kind (three of one denomination and three other cards that don't match). [3]

   c) I look at my hand and notice there are 4 hearts. What is the probability one of them is an ace? [2]

   d) I look at my hand and Have 3 Aces and 3 non-aces. The dealer offers me an opportunity to buy one more card for $10. I know that if I end up with 4 Aces the casino will pay me $1000. Should I buy the additional card? Justify your answer mathematically. [3]

5.

a) I have an “unfair” coin that turns up heads 80% of the time. I flip it 10 times. What is the probability that I get exactly 8 heads? [2]

b) I mix the “unfair” coin in with three other normal “fair” coins and shuffle them around. I pick a random coin out of the four and start flipping it. I flip a head on the first flip. What is the probability that I picked the unfair coin? [3] Show the work that leads to your answer.

6. 40% of Gunn students earned an A last semester in Math. 30% earned an A in Science.

a) If these results are independent of each other, what percent of the students earned an A both in science and in math? [2]

b) what percent of students didn’t get an A in either class? [2]

c) What is the probability that they got an A in science *given that they received an A in math*? [2]

For part d assume that the grades ARE NOT independent any more. Assume 20% earned an A in both classes now.

d) What is the probability if I pick a student at random that they earned an A in at least one of the courses? [2]

7. At Burger IM you choose one of 4 buns, and one of 5 cheeses (or no cheese). Then you can add as many (or as few) toppings as you want. A sign in their store says that there are 40 million different burgers possible. How many different toppings do they have? Show the work that leads to your answer [3].

8. Suppose you are tracing a path along the coordinate plane moving from (0,0) to (9,9) by moving only right and up one unit at a time. If your path is random, what is the probability that you will pass through the point (5, 5)? [3]