9/17/19

Analysis 2019/20 $_iC_u$ to take this quiz: Alan Lee period D

+30/30 11 Awesome!

No calculators. **31 pts. Leave answers in factorial, exponent, or "choose" form.**

1. An ATM code consists of 4 digits, (0 – 9 are possible. It MAY start with 0, and repetition is allowed). [2 each]

a) How many different ATM codes are possible? 10^4 codes

b) How many ATM codes contain at least 1 "6"? $10^4 - 9^4$ codes

$10000 - 9\cdot9\cdot9\cdot9 = 10000 - 6561 = 3439$

c) How many ATM codes contain exactly 2 6's? $\binom{4}{2}\cdot9\cdot9$ codes

$\binom{4}{2}\cdot9\cdot9 = 6\cdot81 = 486$

1
4·9
6·9·9
4·9·9·9

1 2 9 1 6
4 4 6
3 6
1

3439 ✓

2. DJ Dexxter has a record collection of 80 Jazz records and 30 Bollywood records. [2 each]

a) If he chooses three records at random, what is the probability they are all Bollywood?

$$\frac{\binom{30}{3}}{\binom{110}{3}}$$

b) If he chooses three at random, what is the probability that exactly 1 is a Bollywood record and exactly 2 are jazz?

$$\frac{\binom{80}{2}\binom{30}{1}}{\binom{110}{3}}$$

c) If he randomly arranged the 110 records, what is the probability that all of the Bollywood records are next to one another?

1, 2, 3, 4, 5 . . . (80, 81) possible arrangements

$$\frac{81}{\binom{110}{30}}$$

$$\frac{(81!)(30!)}{1(110!)}$$

3. Coach K wants to split his 10 basketball players into two teams of 5. Intuitively he thinks that there are "10 Choose 5" ways of doing this. Unfortunately he is wrong. Explain the error in his thinking. [2] Let us call the players 1, 2, 3 . . . 10.

He is wrong because choosing 1, 2, 3, 4, 5 from the ten players results in the same two teams as choosing 6, 7, 8, 9, 10 from the ten players. Since he is overcounting one invalid choice for every valid choice, we need to divide the original answer by 2 to get $\frac{\binom{10}{5}}{2} = 126$.

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4. My son's baseball team has 12 players on it. [2 each]

a) How many ways are there to pick the 9 players to start the game? $\binom{12}{9}$

b) Once the starters have been selected, how many ways are there to put the 9 starters in a row? $9!$ 362880

c) During the post-game speech, all 12 form a circle. How many ways are there to do that such that Liam and his twin Luke ARE NOT next to each other?

total ways = $11!$

Liam & Luke = $10! \cdot 2$

answer = $11! - 2 \cdot 10! = (11-2)10! = \boxed{9 \cdot 10!}$

5. The new G-pop band "Lederhosen" is giving away stickers at their concert. There are 12 different stickers and they are all free! Hans insists on bringing home at least 10 of the 12 stickers. How many ways are there for him to do so? [2]

$\boxed{\binom{12}{10} + \binom{12}{11} + \binom{12}{12}} = \frac{12 \cdot 11}{2} + 12 + 1 = \boxed{79 \text{ ways}}$

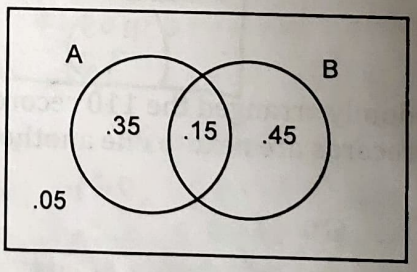
66 12 1

6. Below is a Venn Diagram for the probabilities of two events, A and B occurring. [9]

a. P(A) = 0.5

b. P(B') = 0.4

$P(A' \cap B') =$ 0.05



c. Are A and B mutually exclusive? no Why? because $P(A \cap B) > 0$, meaning both A and B can occur at the same time

d. Are A and B independent? no Why? because $P(A) = 0.5$ and $P(A|B) = \frac{0.15}{0.6} = 0.25$, since $P(A) \neq P(A|B)$, A and B are not independent

e. State the union rule, and show numerically that it is satisfied in this case.

$P(A \cup B) =$ $P(A) + P(B) - P(A \cap B)$ *(use letters here)*

0.95 = $0.5 + 0.6 - 0.15$ (use numbers here) ✓

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