+30/30 4/3

alysis H AtPS Quiz 2 2019/20 Al an Lee has learned the factorials ~~of life.~~

Awesome!

Deggeller / Gleason / Tantod Period: D

1. Use mathematical induction to prove that the given formula works for all positive integers n. [7]

$$1\cdot3+2\cdot4+3\cdot5+\ldots+n(n+2)=n(n+1)(2n+7)/6$$

① Prove for $n=1$

$$1\cdot3=\frac{1\cdot2\cdot9}{6}=3 \checkmark$$

② Assume true for $n=k$

$$1\cdot3+2\cdot4\cdots k(k+2)=k(k+1)(2k+7)/6$$

③ Prove for $n=k+1$ case $\longrightarrow$ Prove $\sum_{i=1}^{k+1} i(i+2)=\frac{(k+1)(k+2)(2k+9)}{6}$

$$1\cdot3+2\cdot4\cdots(k+1)(k+3)$$

$$=\frac{k(k+1)(2k+7)}{6}+\frac{6(k+1)(k+3)}{6}=\frac{(k+1)(2k^2+7k+6k+18)}{6}=\frac{(k+1)(2k+9)(k+2)}{6}$$

$$=\frac{(k+1)(k+2)(2k+9)}{6} \checkmark$$

QED by induction.

2. Use the formula $(n+2)!-n!=n!(n^2+3n+1)$ ✓

to derive a compact expression for: $0!+11(2!)+29(4!)+\ldots+(4m^2+6m+1)[(2m)!]$

Show clear and careful work. [4]

$$(n+2)!-n!=n!(n^2+3n+1)$$

$$0!+11(2!)\cdots(4m^2+6m+1)[(2m)!]$$

$$0!(0^2+3\cdot0+1)+2!(2^2+3\cdot2+1)+4!(4^2+3\cdot4+1)\cdots[(2m)!]((2m)^2+3(2m)+1)$$

$$=(\cancel{2!})-(0!)+(\cancel{4!})-(\cancel{2!})+(\cancel{6!})-(\cancel{4!})\cdots(2m+2)!-(\cancel{2m})!$$

$$=\boxed{(2m+2)!-1} \checkmark$$

TJ

3. Write without factorials and simplify: $\dfrac{[(n+1)!]^2}{n!(n-1)!}$ n^3+2n^2+n [3]

$\dfrac{(n+1)\cancel{(n+1)!}\,(n+1)!}{\cancel{n!}\,(n-1)!} = (n+1)(n) = n(n^2+2n+1) = n^3+2n^2+n$

4. Simplify $\begin{pmatrix} -3 \\ 12 \end{pmatrix}$ 91 [3]

$\dfrac{(-3)(-4)(-5)\cdots(-3-12+1)}{12!} = \dfrac{(-3)(-4)\cdots(-14)}{12!} = \dfrac{(3)(4)(5)\cdots(14)}{12!} = \dfrac{13\cdot 14}{2} = 91$

5. The geometric mean of 6, 36, and 216 is a whole number. Find it. Explain what this number represents.

$\sqrt[3]{6\cdot 36\cdot 216} = \sqrt[3]{6\cdot 6\cdot 6\cdot 6\cdot 6\cdot 6} \Rightarrow 36 = GM$

Geo mean 36 Explain the number that when raised to n, where n = amount of terms, is equal to the product of the terms [3]

6. a) Find the 30th term of the Geometric Sequence with third term 24 and sixth term 3. (Decimal form of answer not needed.)

3	6	9	12	15	18 ,...	30
24	3	$\frac{3}{8}$	$\frac{3}{64}$	$\frac{3}{8^3}$	$\frac{3}{8^4}$.	$\frac{3}{8^8}$

$3/2^{24}$ [3]

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b) Consider the Infinite Geometric Series that corresponds to the sequence described in part "a". Will the Series have a finite sum? Yes or No: Yes. Explain: the common ratio r is less than 1 and more than negative 1 but not 0 so the series will have a finite sum equal to 192 (shown on left) [3]

$S = 96 + 48 + 24 \cdots$

$-\frac{S}{2} = 48 + 24 \cdots$

$\frac{S}{2} = 96 \Rightarrow S = 192$

7. Calculate the sum 11 + 14 + 17 + ... + 761. Write your answer as a product of two numbers. [4]

753

$\frac{753}{3} = 251$ terms

$(251)\left(\frac{761+11}{2}\right) = (251)\left(\frac{772}{2}\right)$

$= 251\cdot 386$

Answer: $251\cdot 386$

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