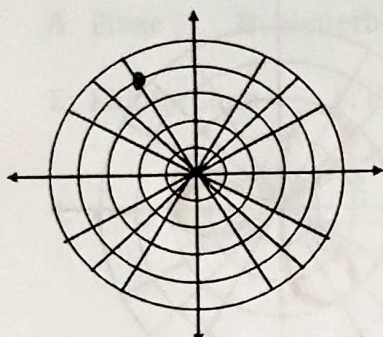$\frac{+29}{31}$ 4

1. Consider the point. $\left(-4, -\frac{\pi}{3}\right)$

a) Plot the point.

b) Find 3 different polar coordinates for this point in the domain $-2\pi \le \theta \le 2\pi$
Give all answers in radians. (1 pt each)

$\left(4, \frac{2\pi}{3}\right)$, $\left(-4, \frac{5\pi}{3}\right)$, $\left(4, -\frac{4\pi}{3}\right)$

c) Convert the point to rectangular coordinates. (2 points)

$\left(-2, 2\sqrt{3}\right)$

2. Show that the polar curve $r = 4\cos\theta - 6\sin\theta$ is a circle by changing it to rectangular. Then find the radius of the circle.

$$\sqrt{x+r^2} = 4r\cos\theta - 6r\sin\theta$$
$$x^2 + y^2 = 4x - 6y + 36y^2$$
$$x^2 - 4x + 4 + y^2 + 6y + 9 = 13$$

Rectangular equation: (3 pts)

$$(x-2)^2 + (y+3)^2 = 13$$

radius (1 pt)

$$\sqrt{13}$$

3. Convert the hyperbola $r = \frac{5}{2\cos\theta + 3\sin\theta}$ into rectangular.

$$2r\cos\theta + 3r\sin\theta = 5$$
$$2x + 3y = 5$$

Rectangular equation: (3 pts)

$$2x + 3y = 5$$

4. Convert the equation: $(x^2 + y^2)^2 = 32xy$ into polar form of the three curves we learned yesterday (polar rose, limacon or lemniscate).

$$r^4 = 32r^2\sin\theta\cos\theta$$
$$r^2 = 32\sin\theta\cos\theta$$
$$r^2 = 16\sin 2\theta$$
$$r = 4\sqrt{\sin 2\theta}$$

~~Reetangular~~ Polur equation: (3 pts)
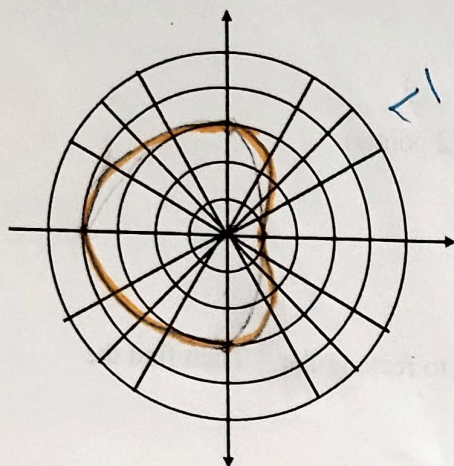
$$r = 4\sqrt{\sin 2\theta}$$

Name of Curve: (1 pt)

___lemniscate___

$$r^2 = 16\sin 2\theta \quad -1$$

16

5. Graph each equation (2 pts each)

$r = 4\sqrt{\cos 2\theta}$
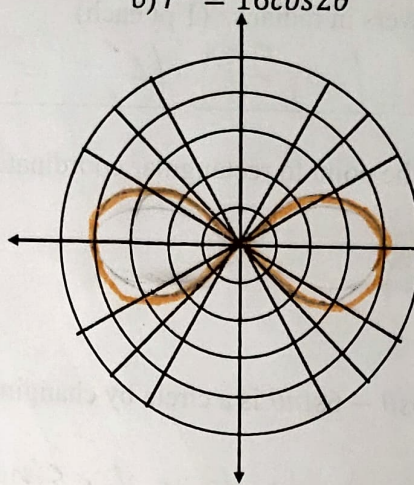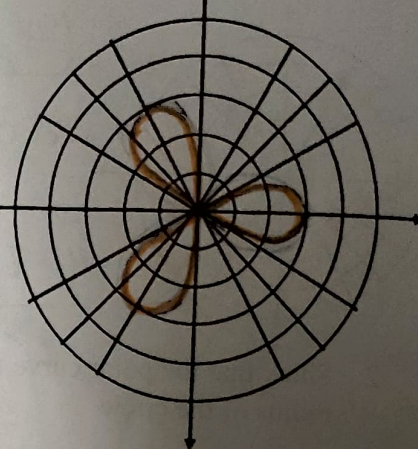
a) $r = 3 - 2\sin\theta$
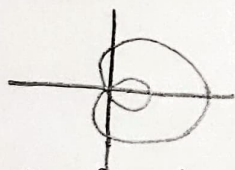
b) $r^2 = 16\cos 2\theta$

c) $r = 3\cos 3\theta$

6. A) Write the equation of an inner loop limacon with a max r-value of 8 on the x-axis.

Equation $\underline{r = 3 + 5\cos\theta}$ (3 pts)

B) For your equation above, name an angle at which it passes through the pole (origin). [2]

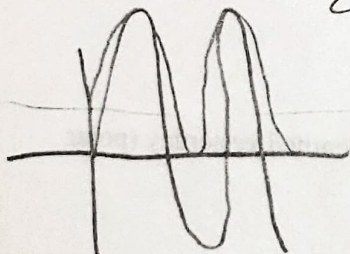$3 + 5\cos\theta = 0$

$5\cos\theta = -3$

$\cos\theta = -\frac{3}{5}$

$\boxed{\arccos\left(-\frac{3}{5}\right)}$

7. The system of equations $r = 4\sin\theta$ and $r = 2 + 2\sin\theta$ has exactly two geometric solutions. Find both of them. Feel free to graph if you'd like but you don't need to.
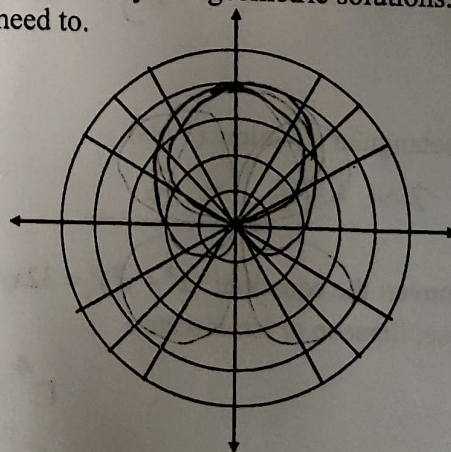

$2 + 2\sin\theta = 4\sin\theta$

$2 = 2\sin\theta$

$\sin\theta = 1$

$\theta = 90°$

Solutions: $\underline{(0, 0°)}$, $\underline{(4, 90°)}$ [3]