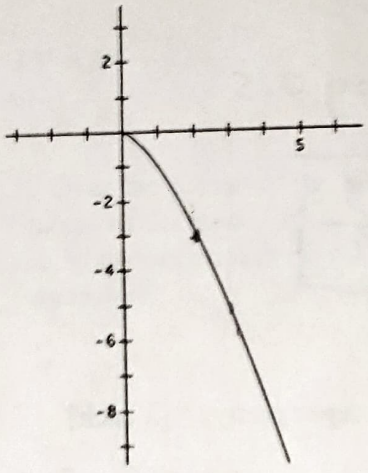
Analysis Quest 2019/20 I'm gonna power through this one Alan Lee

25 +23
25

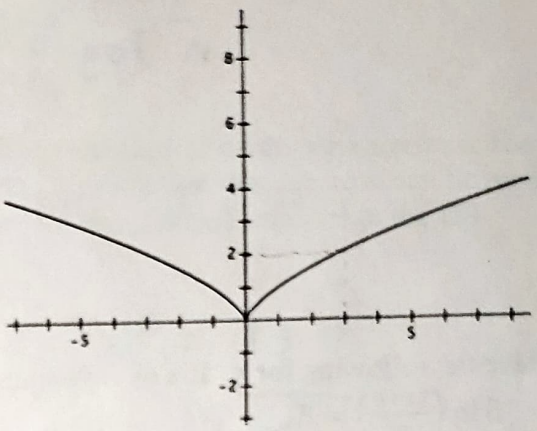
+48
48

No Calculator Page.

1. Below are the graphs of two power functions with rational exponents. Write a possible equation for each. There are many correct answers. [3 each]

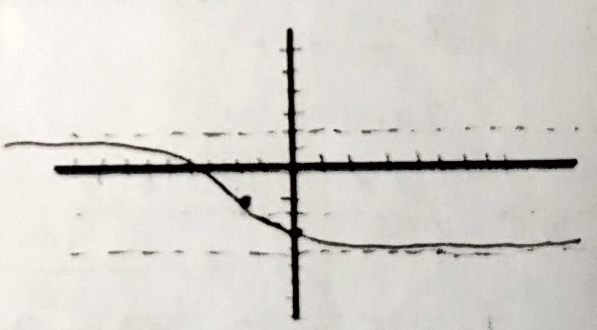


a $y = -x^{\frac{2}{2}}$



b $y = x^{\frac{2}{3}}$

2. Sketch a graph of the following transformed logistic function clearly noting its y intercept, asymptotes and point of inflection. $f(x) = \frac{6}{1+5e^{-x}} - 4$



$\frac{6}{1+5e^{x}} = 3$

$\frac{1}{1+5e^{x}} = \frac{1}{2}$

$5e^{x} = 1$

$e^{x} = \frac{1}{5}$ $\quad -\ln 5$

y intercept $(0, -3)$

asymptotes: $y = 2, y = -4$

point of inflection (as an ordered pair) $(-\ln 5, -1)$

B

3. The population of Venice is shrinking by 4% every year. How long will it take until the population is cut in half?

n years: $(0.96)^n = 0.5$

$n \log 0.96 = \log 0.5$

$$n = \boxed{\frac{\log 0.5}{\log 0.96}}$$

4. Solve the following for x. It's ok if your answers aren't super pretty. [3 each]

a) $3 + 5\ln\left(\frac{2x+8}{7}\right) = 5$

$5\ln\left(\frac{2x+8}{7}\right) = 2$

$\ln\left(\frac{2x+8}{7}\right) = \frac{2}{5}$

$\frac{2x+8}{7} = e^{\frac{2}{5}}$

$2x+8 = 7e^{\frac{2}{5}}$

$2x = 7e^{\frac{2}{5}} - 8$

$$x = \boxed{\frac{7e^{\frac{2}{5}}}{2} - 4}$$

b) $5^x = 2^{3x-5}$

$x \log 5 = (3x-5)\log 2$

$x\log 5 = 3x \log 2 - 5\log 2$

$x\log 5 = x \log 8 - 5\log 2$

$5\log 2 = x \log \frac{8}{5}$

$$x = \boxed{\frac{5\log 2}{\log 8 - \log 5}}$$

Calculator Section: Name again Alan Lee

Turn in the No-calculator section before using your calculator on this section.
Round all final answers to at least 2 decimal places.

Potentially Useful formulas:

$$FV = R\frac{(1+i)^n - 1}{i} \qquad PV = R\frac{1-(1+i)^{-n}}{i}$$

1. Over the course of the last 15 years, the percentage of adults who smoke in the United States has decreased from 20.9% to 13.7%. Assume that this decrease can be modeled by yearly exponential decay. At what rate is the percentage of smokers in the US decreasing?

$20.9(x)^{15} = 13.7$

$x = \sqrt[15]{\frac{13.7}{20.9}}$ $\quad 1-x = 0.02772\ldots$

$x = 0.972236\ldots$

2.776%

2. You win the lottery!!! The state of California offers you two choices: A lump sum payment of $10 million dollars right now OR $1 million a year for 15 years. Assume that you can get an average yearly rate of return of 3% on your money. Which of these is a better deal financially? Make a mathematical argument using Present and/or Future values.

Present Value

Option ① = $10 million

Option ② = $1000000\frac{1-(1+0.03)^{-15}}{0.03} = \$11,937,935.08$

Option ② PV > option ① PV, so **Option ② is better**

3. Felisha is planning for retirement and knows that she will need a million dollars in 18 years to live comfortably. Assuming that she can get an average yearly rate of return of 4% on her money, how much should she be putting away **each month** for her to reach her goal in 18 years?

18 years = 216 months

Future Value

$1000000 = R\frac{\left(1+\frac{0.04}{12}\right)^{216}-1}{\frac{0.04}{12}}$

$1000000 = 315.542R$

$R \approx \$3168.64$

$3168.64

4a) Your buddy Chachi owes you $2000, but is paying you back $75 each month. Because you are a swell dude/dudette, you are only charging him 1% yearly interest. How many months will it take Chachi to pay you back?

Present Value

$$2000 = 75\,\frac{1-\left(1+\frac{0.01}{12}\right)^{-n}}{\frac{0.01}{12}}$$

$$0.02222\overline{2} = 1-\left(1+\frac{0.01}{12}\right)^{-n}$$

$$\left(1+\frac{0.01}{12}\right)^{-n} = 0.977\overline{7}$$

$$0.99916^{n} = 0.977\overline{7}$$

$n \log 0.99916 = \log 0.977\overline{7}$

$n = \frac{\log 0.977\overline{7}}{\log 0.99916}$

$n \approx 26.978$

round to 2

$\rightarrow 26.98$

26.98 months

4b) Chachi has a theory: If he can double his payment every month (to $150), he will cut the time that it will take him to pay it back **in half**. Justify mathematically whether you agree or disagree with Chachi, clearly showing your calculations.

Present Value

$$2000 = 150\,\frac{1-\left(1+\frac{0.01}{12}\right)^{-n}}{\frac{0.01}{12}}$$

$$0.0\overline{1} = 1-\left(1+\frac{0.01}{12}\right)^{-n}$$

$$0.9\overline{8} = 0.99916^{n}$$

$$\frac{\log 0.98\overline{8}}{\log 0.99916} = n$$

$n \approx 13.4135$

$13.4135 < \frac{1}{2}(26.98) = 13.49$

as such, it will take him less than half the time

$\therefore$ I disagree.

5. Consider a logistic equation in the form $y = \frac{A}{1+Be^{-kt}}$. The carrying capacity of the population is 100. The initial population (at t = 0) is 10, and after 4 years, the population increased to 20. Solve for A,B, and k clearly showing your work.

A= 100

B= 9

k= 0.2027

$\frac{100}{1+B} = 10$

$B = 9$

$20 = \frac{100}{1+9e^{-4k}}$

$e^{-4k} = \frac{4}{9}$

$e^{4k} = \frac{9}{4}$

$4k = \ln \frac{9}{4}$

$k = \ln \frac{9}{4} / 4$

$1+9e^{-4k} = 5$

$9e^{-4k} = 4$