GATM Quiz # 1 2019/20

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Your period D

+20/20

1. For this problem you will be considering whether the set of positive and negative odd numbers forms a mathematical ***group*** under addition.

State the four requirements for being a mathematical group, and conclude whether the set of numbers above (under addition) passes that requirement.

a) requirement closure passes? no odd + odd = even!

b) requirement identity passes? no no $\emptyset$ → identity

c) requirement inverse passes? no no inverse if there is no identity

d) requirement associativity passes? yes $(a+b)+c = a+(b+c)$

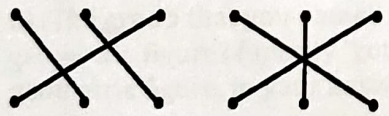
So, is it a group under addition? no

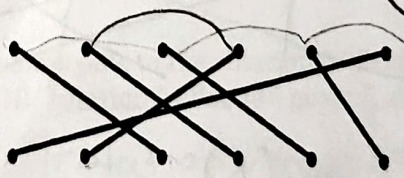
2. The rotation group of a hexagon and the reflection group of a equilateral triangle both have 6 elements, though they are NOT isomorphic.

In a sentence or two, give a solid reason that you know they are not isomorphic. Use specific language.

The second (triangle) group is not commutative, For example $r120° \circ ref(A)$ $\left(\begin{smallmatrix}B\\C\end{smallmatrix}\triangle A\right) = \begin{smallmatrix}A\\C\end{smallmatrix}\triangle B$, while $ref(A) \circ r120° \left(\begin{smallmatrix}B\\C\end{smallmatrix}\triangle A\right) = \begin{smallmatrix}B\\A\end{smallmatrix}\triangle C$. However, the hexagon group is commutative since $ra° \circ rb° = rb° \circ ra°$.

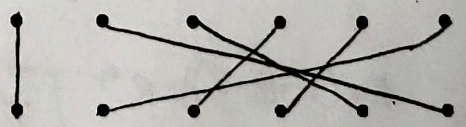
3. Below are two members of the 6 post group

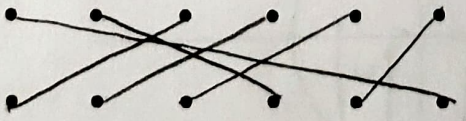
Q: 

R: 

a) State the period of Q 6

b) State the period of R 4

c) Draw Q o R (Q snap R where R goes first) to the right: 

d) Draw the inverse of R 

e) How many elements total are there in the six post group? 720

Would element R generate the entire 6 post group? No Explain how you know, use the back too. The period of R is four, so using R alone would only generate at most 4 distinct elements. Snapping R on itself 3 times gives $(R \circ (R \circ (R \circ (R)))) = I$, as mentioned in the period above. Thus, we [illegible] with

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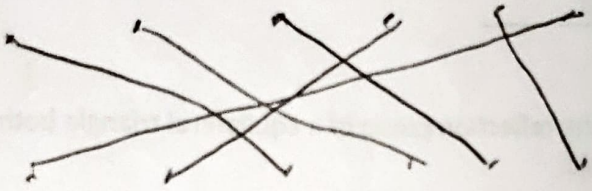
4 snaps would return the resulting element into R again, rather than creating a new element different from the 4 already discovered. Thus, since R can only generate 4 distinct elements; it cannot be a generator for the entire 6-post group, which has $6! = 720$ elements.

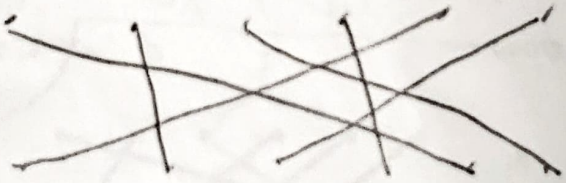
Fig: 1 Showing the 'cycle' of R^4 and R^5

R



$R^5 = R(R \circ (R \circ (R \circ R)))$
$= R \circ I = R$

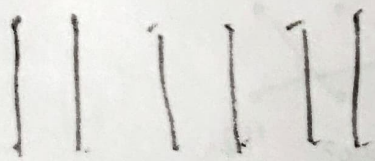
$R \circ R = R^2$



$R^6 = R \circ (R^5)$
$= R \circ R \circ I$
$= R \circ R = R^2$

$R \circ (R \circ R) = R^3 = inv(R)$

$R \circ (R \circ (R \circ R)) = R^4 = I$



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cycles!