Analysis Unit4 Vectors and Parametrics Quiz1
Deggeler/Gleason/Tantod (30 points)

Vectorious Student: Alan Lee

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Calculator Needed for #1f, 2, 4

1. Given $\vec{u}=\langle 2,4\rangle$, $\vec{v}=\langle -3,-5\rangle$, and $\vec{w}=\langle 3,-7\rangle$, find the following (a-d are 1 point, e is 2 points, f-h 3 pts each):

a) $|\vec{u}|$ $2\sqrt{5}$

b) $2\vec{u}+\vec{w}$ $\langle 7,1\rangle$

c) $\vec{u}\cdot\vec{w}$ -22

d) $\hat{v}$, a unit vector in the direction of $\vec{v}$ $\left\langle \frac{-3\sqrt{34}}{34}, \frac{-5\sqrt{34}}{34} \right\rangle$

e) a vector orthogonal to $\vec{w}$ $\langle 7,3\rangle$

f) the angle between $\vec{w}$ and $\vec{v}$ (in degrees)
Round your answer to 3 decimal places.

$w\cdot v = 26 = \sqrt{34}\sqrt{58}\cos\theta$

$\cos\theta = 0.585$

$\theta = 54.162°$

g) vector $proj_{\vec{u}}\vec{w}$ $\langle -2.2, -4.4\rangle$

$|w| = \sqrt{58}$

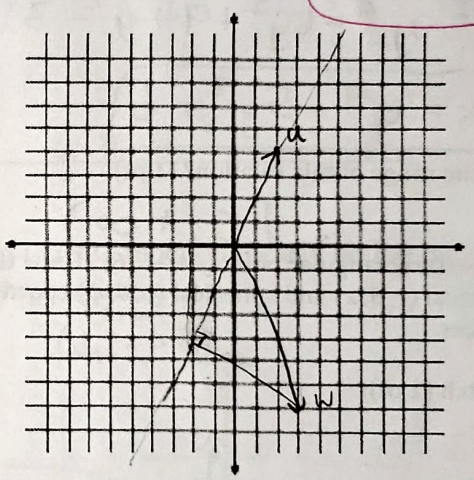
$\vec{u}\cdot\vec{w} = -22 = 2\sqrt{5}\cdot\sqrt{58}\cos\theta$

$\theta = 130.236$

$\cos\theta = -0.645\ldots$

$|w|\cos\theta = -4.91935$, $\left\langle \frac{\sqrt{5}}{5}, \frac{2\sqrt{5}}{5} \right\rangle$

h) sketch and label $\vec{u}$, $\vec{w}$, and vector $proj_{\vec{u}}\vec{w}$ on the axes:



labels −½

2. Calvin is on the roof of his house (15 feet tall) with a pile of water balloons and a giant slingshot. The slingshot is fixed to launch water balloons at 24 degrees above horizontal, but he can change the velocity of the balloons by how much he pulls the rubber band back. He sees a fire hydrant (which is 4 feet tall), across the street, 120 feet away. How fast does a water balloon need to leave the slingshot in order to hit the top of the fire hydrant? *Round your answer to 3 decimal places.* (4 pts)

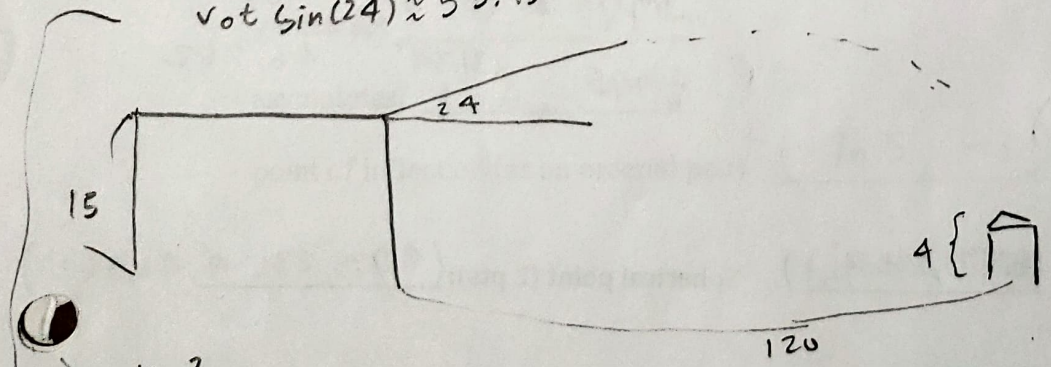
$-16t^2 + v_0\sin(24)t + 15 = 4$

$v_0 t\cos(24) = 120$

$v_0 t\sin(24) \approx 53.43$

vert: $-16t^2 + v_0\sin(24)t + 15$

horiz: $v_0 t\cos(24)$



$-16t^2 + 15 + 53.43 = 4$

$-16t^2 = -64.43$

$t \approx 2.007$

$2.007\,V_0\cos 24 = 120$

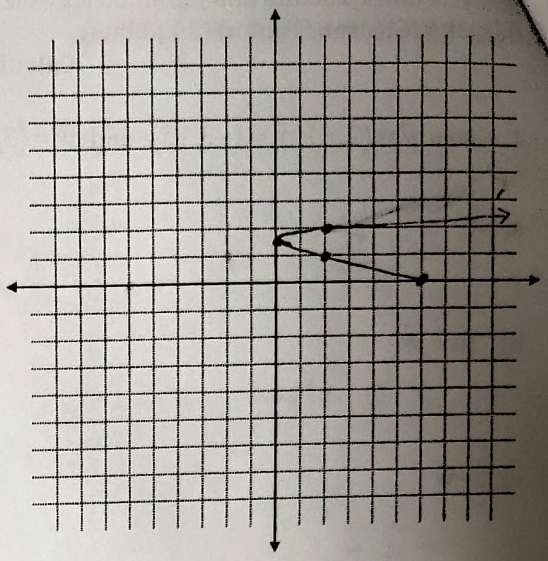
$V_0 \approx 65.4599$

$V_0 = 65.46$

18½/20

3. Given the following set of parametric equations:

$$x = t^2 + t$$
$$y = \sqrt{t+3} \qquad \text{for } t: [-3, \infty)$$



a) Sketch the set of parametric equations on the grid to the right. Label at least 3 points, using $t = -3, -2$, and -1. (3 pts)

b) Eliminate the parameter and write x as a function of y. (2 pts)
No need to simplify.

$y^2 = t + 3$

$x = (y^2 - 3)^2 + (y^2 - 3)$

$x = y^4 - 6y^2 + 9 + y^2 - 3$

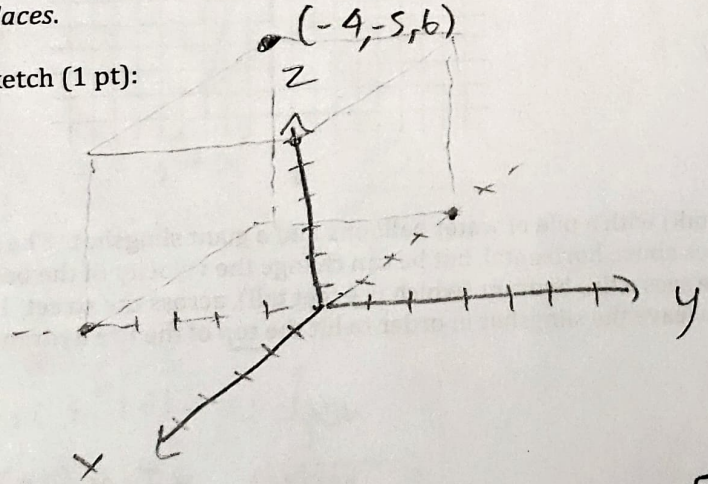
$\boxed{x = y^4 - 5y^2 + 6}$

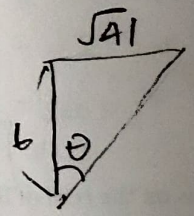
c) What is the range of this relation? (1 pt)

$[0, +\infty)$

4. Draw the 3D rectangular point $(-4, -5, 6)$. Draw the box or vectors in your sketch. Then convert the point into both cylindrical (r, θ, z) and spherical (ρ, θ, ϕ) coordinates. Give all angles in degrees, and *round your answers to 3 decimal places.*

Sketch (1 pt):





$(-4, -5) \Rightarrow (r, \theta)$

$(\sqrt{41}, 231.34)$

$\sqrt{16+25+36}$

$\sqrt{77}$ $\quad \theta$ $\quad \theta_2$

8.775 $\quad$ 231.34° $\quad$ 46.862

Cylindrical point (2 pts): $(6.403, 231.34°, 6)$ Spherical point (2 pts): $(8.775, 231.34°, 46.862°)$

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