Analysis S1 Final Exam REVIEW SHEET 4

1. Show that
$$\binom{10}{1} \binom{9}{2} \binom{7}{3} = \binom{10}{4} \binom{6}{3} \binom{3}{2} = \binom{10}{2} \binom{8}{4} \binom{4}{1}$$

2. Find each of the following

a)
$$2\left[\binom{5}{0} + \binom{5}{1} + \binom{5}{2}\right]$$

b)
$$2 \begin{bmatrix} 6 \\ 0 \end{bmatrix} + \begin{pmatrix} 6 \\ 1 \end{pmatrix} + \begin{pmatrix} 6 \\ 2 \end{bmatrix} + \begin{pmatrix} 6 \\ 3 \end{pmatrix}$$

c)
$$\binom{4}{0} - \binom{4}{1} + \binom{4}{2} - \binom{4}{3} + \binom{4}{4}$$

d)
$$\binom{100}{0} - \binom{100}{1} + \binom{100}{2} - \binom{100}{3} + \dots - \binom{100}{99} + \binom{100}{100}$$

3. Find each sum.

a)
$$24 + 32 + 40 + ... + 800$$

d) The 30th row of Pascal's Triangle

e)
$$6 - \frac{6}{2^2} + \frac{6}{2^4} - \frac{6}{2^6} + \dots + \frac{6}{2^{188}} =$$

- 4. Write each series from the previous question in Sigma Notation.
- 5. What are the key identifiers of a proof by Mathematical Induction?
- 6. For the set: {2, 6, 18, 54}, calculate...
 - a) the arithmetic mean
 - b) the geometric mean
- 7. Find r and s such that $\frac{r!}{s!} = \frac{1}{20.19.18.17}$
- 8. What is the common difference of an arithmetic sequence whose first term is 2, and all 20 terms add to 300?

- 9. Find the 1st, 5th, and 10th terms of the expansion of $(4x-3y)^{17}$
- 10. Use the patterns that we found in Pascal's Triangle to write each of the following as a single term or binomial coefficient.

a)
$$\binom{n}{k} + \binom{n}{k+1}$$

b)
$$\binom{n-1}{k-1} + \binom{n-1}{k} + \binom{n}{k+1} + \binom{n+1}{k+2}$$

c)
$$\binom{87}{0} + \binom{88}{1} + \binom{89}{2} + \binom{90}{3} + \dots \binom{143}{56}$$

d)
$$\binom{n+10}{n+10} + \binom{n+11}{n+10} + \binom{n+12}{n+10} + \dots + \binom{n+41}{n+10}$$

e)
$$\binom{n+10}{0} + \binom{n+10}{2} + \binom{n+10}{4} + \dots + \binom{n+10}{n+8} + \binom{n+10}{n+10}$$

11. Name the following 3 –D surfaces:

a)
$$\frac{x^2}{7} - \frac{y^2}{12} = \frac{z^2}{7}$$

b)
$$\frac{x^2}{7} - \frac{y}{12} = \frac{z^2}{7}$$

c)
$$\frac{x^2}{7} = \frac{z^2}{7} + 11$$

d)
$$\frac{x^2}{7} = \frac{z^2}{7} + y + 11$$

- 12. How much more money would Joe have after 10 years if he invested his \$4000 dollars (@ 4% annual interest) if his bank compounded continuously rather than monthly?
- 13. A population of emus doubles every 13 years. How long will it take the population to triple?
- 14. Joey is playing a game with a 10 sided die. If he rolls a multiple of 3 his brother gives him 1 dollar. If he rolls a prime number he has to pay his brother 3 dollars. This of course is not a fair game. Make it a fair game by creating a payout/payment for rolling a "10".
- 15. Write a parametric equation for an ellipse with center (8, 2) with "x radius" = 4 and "y radius" = 5. Explain what would happen if you reversed sine and cosine in your equation.
- 16. In our classroom we have 36 seats (9 groups of 4). I randomly assign 36 students to the seats. What is the probability you are in a group with your best friend?