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Questions 1-4 are Multiple Choice. Circle the best answer [2 pts each]

1. Which of the following expressions is equivalent to $\binom{23}{10}$?

I. $\binom{24}{11} - \binom{23}{11}$

II. $\binom{22}{9} + \binom{22}{10}$

III. $\binom{21}{8} + 2\binom{21}{9} + \binom{21}{10}$

~~a) I only~~

~~b) II only~~

~~c) I and II only~~

d) I, II, and III

~~e) II and III only~~

2. Given two numbers a and b, the geometric mean of the two numbers is _____ greater than the arithmetic mean of the two numbers.

a) always

~~b) only in the case of $a > b$~~

~~c) only in the case of $a < b$~~

d) usually

~~e) never~~

3. Which of the following is the best example of Mathematical Induction?

a) I get paid every week, and I have money in my bank account right now. As long as I spend less money than I make each week, I know that I'll have money in my bank account at the end of next year.

b) $(x+1)(x+2)$ will always be even, because $(x+1)$ or $(x+2)$ must be even.

c) All dogs are mammals, and all mammals breathe oxygen. Therefore, dogs breathe oxygen.

d) If $A + 5 = B$, then $A = B - 5$.

e) Thor should obviously be stronger than Captain America. But seeing as how Marvel randomly scales relative power levels between scenes, there will probably be a fight in phase 4 where Spider-Man, Ant-Man, and Shang-Chi all fight toe to toe and none of them can beat the others.

4. The expression $\frac{(n+2)!}{n! + (n+1)!}$ can be simplified and expanded into the form $\frac{an^2 + bn + c}{dn + e}$.

What is the value of $a + b + c + d + e$?

a) 5

b) 6

c) 7

d) 8

e) 9

-2

Free Response. Clearly show all your work to receive full credit!

5. $F_{45}F_{114} + F_{44}F_{113} = F_n$. Find the value of n . [1 pt]

$$F_{159} + F_{157} = F_{158}$$

$$n = 158$$

$$\begin{array}{r} 119 \\ 93 \\ \hline 159 \end{array} \quad \begin{array}{r} 113 \\ 94 \\ \hline 157 \end{array}$$

6. Evaluate: [2 pts]

$$\sum_{k=2}^{\infty} 1000(0.3)^k$$

$$(0.3)(0.3)$$

$$1000 \cdot 0.09 = 90.00$$

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$$\frac{90}{0.7}$$

$$1000(0.3)^2 + 1000(0.3)^3 + 1000(0.3)^4 \dots$$

$$S_{\infty} = \frac{a_1}{1-r}$$

$$1000[(0.3)^2 + (0.3)^3 + (0.3)^4 \dots]$$

$$S_{\infty} = \frac{0.09}{0.7} \cdot \frac{1000}{1} = \frac{90}{0.7}$$

7. Consider the following sequence, where 7 is the first term:

$$7, 8, 10, 13, 17, 22, \dots$$

a) Find the 7th term. [1 pt]

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b) Find the 10th term. [1 pt]

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c) Find an expression for the n^{th} term. [2 pt]

$$(n-1)^{\text{th}} \text{ term} + (n-1)$$

term	value
1	7
2	8
3	10
4	13
5	17
6	22
7	28
8	35
9	43
10	52

8. Find the 15th term of the binomial expansion of $(-x + 3y)^{36}$. (Leave your answer as a product of choose numbers and exponents – don't multiply it out). [2 pts]

$$(-x + 3y)^{36}$$

$$\binom{36}{14} (-x)^{22} (3y)^{14}$$

$$\begin{array}{c} 1 \\ 1 \quad 2 \quad 1 \\ 1 \quad 3 \quad 3 \quad 1 \end{array} \quad \begin{array}{c} 0 \\ 1 \\ 2 \\ 3 \end{array} \quad \begin{array}{c} \binom{0}{0} \\ \binom{1}{0} \binom{1}{1} \\ \binom{2}{0} \binom{2}{1} \binom{2}{2} \\ \binom{3}{0} \binom{3}{1} \binom{3}{2} \binom{3}{3} \end{array}$$

Questions 9-10: Multiple Choice again! Choose the best answer. [2pts each]

9. The polar graph of $r = 4 - 9 \cos \theta$ is a _____.

- ☒ a) rose curve
☐ b) dimpled limaçon
☒ c) limaçon with an inner loop
☐ d) convex limaçon
☒ e) potato

10. Consider the graph of $r = 4 \sin 7\theta$. Which is a line of symmetry?

I. $\theta = 0$

II. $\theta = \frac{3\pi}{2}$

III. $\theta = \frac{13\pi}{14}$



- ☒ a) I only
☐ b) II only
☐ c) III only
☒ d) I, II, and III
☒ e) II and III

★ 11. Consider the polar graph $r = 2 \cos \theta + 2 \sin \theta$

a) This graph goes through the pole. What is the smallest positive value of θ at which this occurs? [1]

~~0~~ -1

b) Convert the equation to rectangular form. [2 pts]

$$r = 2 \cos \theta + 2 \sin \theta$$

$$r^2 = 2x + 2y$$

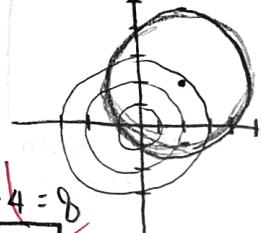
$$r^2 = 2r \cos \theta + 2r \sin \theta$$

$$x^2 + y^2 = 2x + 2y$$

$$x^2 - 2x + 4 + y^2 - 2y + 4 = 8$$

$$(x-2)^2 + (y-2)^2 = 8$$

$$\begin{array}{r} 2,8 \\ 2,8 \\ \hline 22,4 \\ 56 \\ \hline 784 \end{array}$$



c) What shape is the graph? [1 pt]

circle

12. Convert the given polar equation into rectangular, and solve for y. [2 pts]

$$\tan \theta = \sqrt{3}$$

$$\frac{r}{r} \cdot \frac{\sin \theta}{\cos \theta} = \sqrt{3}$$

$$\frac{r \sin \theta}{r \cos \theta} = \sqrt{3}$$

$$\frac{y}{x} = \sqrt{3}$$

$$y = \sqrt{3} x$$

13. Convert the given rectangular equation into polar, and solve for r. [3 pts]

$$(x+4)^2 + (y-1)^2 = 17$$

$$x^2 + 8x + 16 + y^2 - 2y + 1 = 17$$

$$x^2 + y^2 + 8x - 2y = 0$$

$$r^2 = -8r \cos \theta + 2r \sin \theta$$

$$r = -8 \cos \theta + 2 \sin \theta$$

-2

14. Consider the polar graphs

$$r = 4$$

$$r = 8 \sin 3\theta$$

a) Draw both of the graphs in the space on the right. [2 pts]

b) Find all the points of intersection of the graphs. [3 pts]

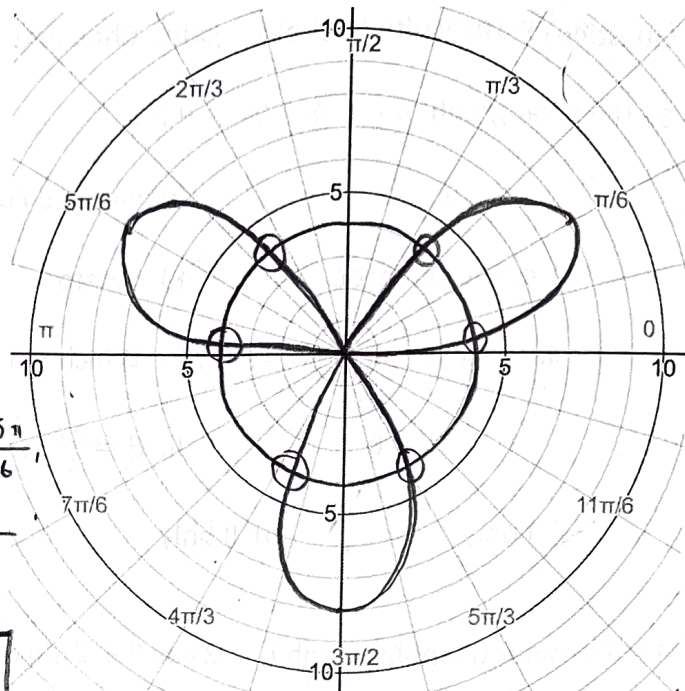
$$4 = 8 \sin 3\theta$$

$$\frac{1}{2} = \sin 3\theta$$

$$3\theta = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{13\pi}{6}, \frac{17\pi}{6}, \frac{25\pi}{6}, \frac{29\pi}{6}$$

$$\theta = \frac{\pi}{18}, \frac{5\pi}{18}, \frac{13\pi}{18}, \frac{17\pi}{18}, \frac{25\pi}{18}, \frac{29\pi}{18}$$

$$\left(4, \frac{\pi}{18}\right) \left(4, \frac{5\pi}{18}\right) \left(4, \frac{13\pi}{18}\right) \left(4, \frac{17\pi}{18}\right) \\ \left(4, \frac{25\pi}{18}\right) \left(4, \frac{29\pi}{18}\right)$$



15. For each of the following, name the graph. No need to worry about orientation. [1 pt each]

a) $y^2 + z^2 = 12x$ elliptic paraboloid
 $y^2 + z^2 - 12x = 0$

$$x^2 + y^2 + z^2 = 1$$
 ellipsoid

b) $15z^2 - x^2 = y^2$ elliptic cone
 $15z^2 + y^2 - x^2 = 0$

$$x^2 + y^2 - z^2 = 1$$
 1 sheet

$$x^2 + y^2 - z^2 = 0$$
 elliptic cone

c) $x^2 - y^2 + 4z^2 = -8$ hyperboloid of 2 sheets
 $x^2 - y^2 + 4z^2 + 8 = 0$

$$x^2 + y^2 - z^2 = -1$$
 2 sheets

$$x^2 + y^2 - z^2 = 0$$
 elliptic paraboloid

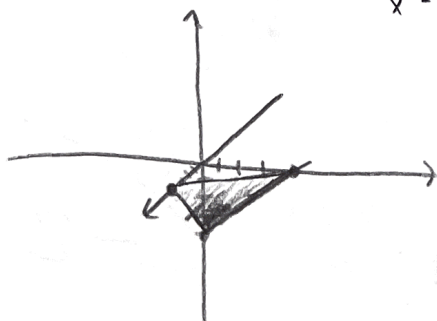
d) $-x^2 - y^2 + z^2 - 8 = 0$ hyperbolic paraboloid
 $z^2 - x^2 - y^2 - 8 = 0$

$$x^2 - y^2 - z^2 = 0$$
 hyperbolic paraboloid

e) $x + y - z = 4$ plane

f) $\frac{x^2}{16} - \frac{z^2}{16} = y$ hyperbolic paraboloid

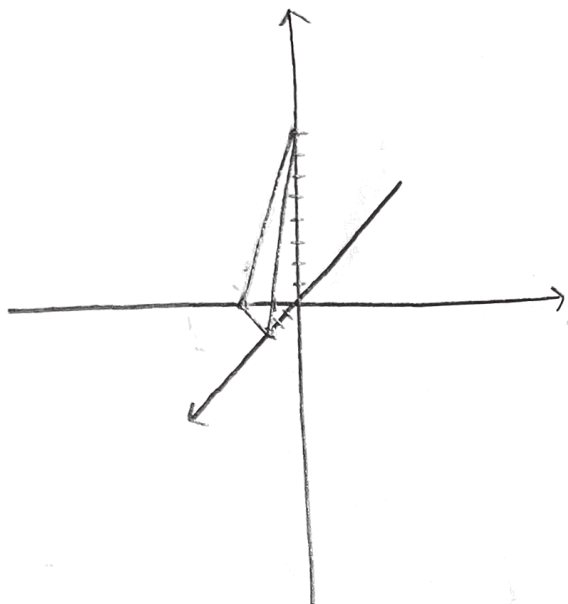
$$x^2 - z^2 - 16y = 0$$



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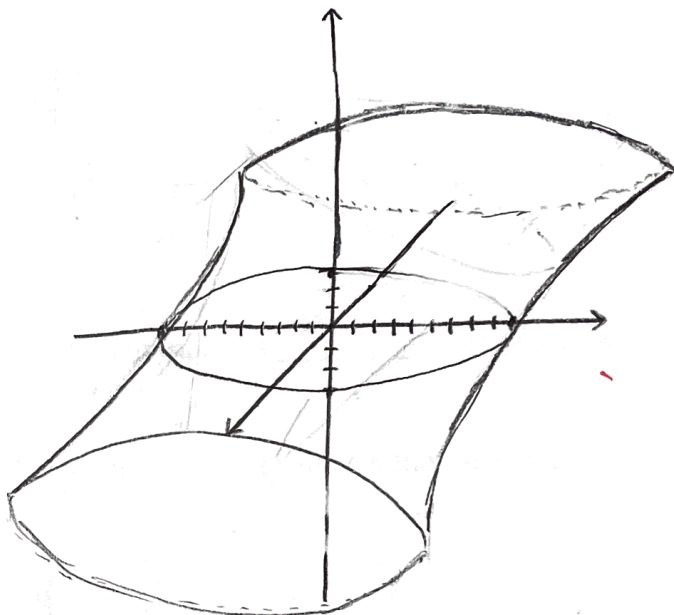
16. Sketch the following graphs in 3D. [3 pts each]

a) $3x - 4y + z = 8$

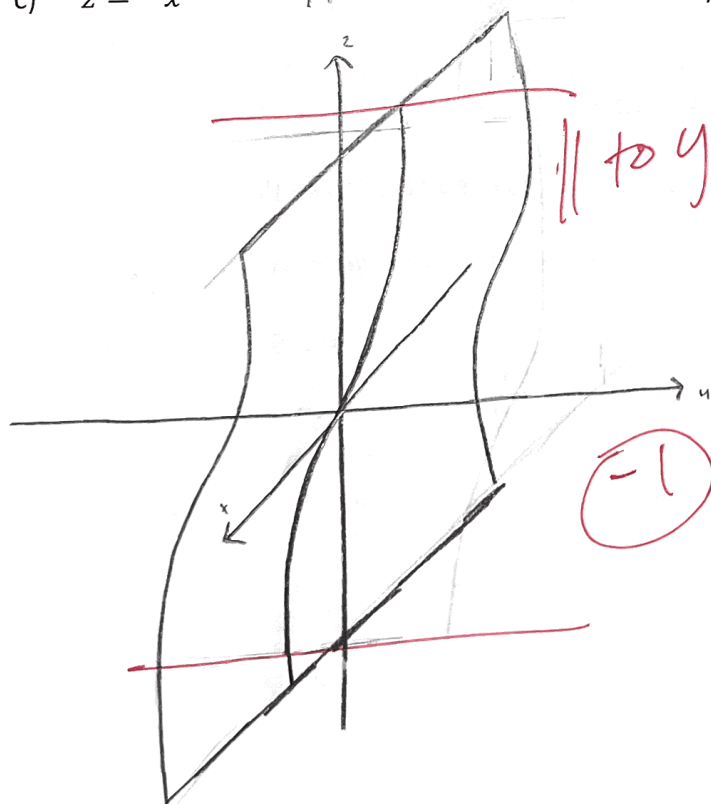
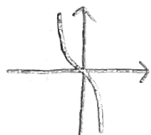


b) $-\frac{x^2}{9} + \frac{y^2}{81} + \frac{z^2}{9} = 1$

$$\frac{y^2}{81} + \frac{z^2}{9} - \frac{x^2}{9} = 1$$



c) $z = -x^3$



d)

$$x^2 + z^2 + y = 0$$

$$0 = x^2 + y + z^2 - 4$$

$$\frac{x^2}{4} + \frac{z^2}{4} + \frac{y}{4} = 1$$

$$0 = z^2 + y - 4$$

$$-z^2 + 4 = y$$

$$0 = x^2 + y - 4$$

$$-x^2 + 4 = y$$

